Artificial Gravity:
Why Centrifugal Force is a Bad Idea

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The concept of centrifugal force is taught in elementary science education, sometimes even before Newton's Laws are introduced. However, centrifugal force fails to explain orbit, weightlessness, weight, or "artificial gravity" in a way that is consistent with Newtonian physics. In terms of the operative physical forces, centrifugal is to centripetal as Ptolemy is to Newton. The centrifugal-force point of view invokes fictitious causes of illusory motions that ultimately lead to contradiction, misconception, and confusion. A proper understanding of the actual forces acting on a moving body in a rotating frame of reference is essential to the design of safe and comfortable artificial-gravity habitats.

I. Nomenclature

x, y, z	Cartesian coordinates of an object in a rotating frame of reference
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit basis vectors parallel to the x, y, z axes
$\mathbf{r}, \mathbf{v}, \mathbf{a}$	position, velocity, and acceleration vectors relative to the rotating frame
X, Y, Z	Cartesian coordinates of an object in a non-rotating inertial frame
$\mathbf{I}, \mathbf{J}, \mathbf{K}$	unit basis vectors parallel to the X, Y, Z axes
$\mathbf{R}, \mathbf{V}, \mathbf{A}$	position, velocity, and acceleration vectors relative to the inertial frame
$\mathbf{\Omega}$	rate of rotation of x, y, z relative to X, Y, Z as radians per unit time
t	elapsed time
$\mathbf{A}_{cent}$	centripetal acceleration vector $= -\Omega^2 \cdot \mathbf{r}$
$\mathbf{A}_{Cor}$	Coriolis acceleration vector $= 2 \cdot \mathbf{\Omega} \times \mathbf{v}$
$\mathbf{F}$	force vector
G	universal gravitational constant $= 6.674 \times 10^{-11} \cdot \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$
m	mass

II. Introduction

Centrifugal force is a familiar concept, first encountered in elementary school science classes, and carried by scientists and engineers all the way through Ph.D. theses, laboratory work with centrifuges, and the design and analysis of various rotating structures – including artificial-gravity habitats. But, just because a concept is familiar, or learned and preserved from an early age, does not make it a good idea. Moreover, such early lessons, conveyed by adults to children in an atmosphere of trust (or authority), may be the most liable to assume the aura of dogma and the most resistant to unlearning.

Consider, for example, this introduction posted by Study.com in its "Centrifugal Force Lesson for Kids" [1]: "centrifugal force: the energy of a moving object in a circle trying to stay in a straight line when it cannot." This confused definition, which conflates the concepts of force and energy, will subsequently need to be unlearned if the students are to understand anything about Newtonian physics and dynamics, or energy supply and demand. Would

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it not be better to teach Newton's Laws from the beginning? What could be simpler than his Second Law of Motion, $\mathbf{F} = m \cdot \mathbf{A}$, along with the concept that acceleration is any change from constant-speed straight-line motion? Perhaps a confused definition is nearly inevitable for a concept that should not be taught as anything other than an illusion.

In general science education regarding the solar system, it has become somewhat dogmatic that Ptolemy's Earth-centered concept was "wrong" and Copernicus's Sun-centered concept is "right." Nevertheless, the concept of centrifugal force persists even though it is based on an essentially Ptolemaic, non-Copernican, non-Newtonian frame of reference. Consistency seems to demand that if Ptolemy was "wrong," then centrifugal force is "wrong."

This paper argues that "centrifugal force" is an inherently confused concept that introduces an unnecessary phantom that contradicts our most reliable physical theories. "Centrifugal force" obscures a simpler and more consistent understanding of the essential dynamics and is therefore not a good mental model for artificial-gravity habitat design. The argument begins with statements that I expect should be non-controversial for many readers, but builds toward concepts that may be unfamiliar, unconventional, yet useful for the understanding, analysis, and design of artificial-gravity habitats. The line of reasoning proceeds through the following sections:

   III. Philosophy of Science: Good, Bad, Right, Wrong.
   IV. Orbit: Centrifugal Force Contradicts Newton.
    V. Weightlessness: Centrifugal Force Cannot Account For It.
   VI. Weight is Due to the Upward Push of the Floor (not the Downward Pull of Gravity).
  VII. Weight in a Rotating Space Habitat is Due to Centripetal Force.
 VIII. The Centrifugal Force View Renders Coriolis Force Backward.
   IX. Conclusion.

III. Philosophy of Science: Good, Bad, Right, Wrong

This paper must begin with a discussion of what it means by "good" and "bad" in science and engineering. It will not attempt to argue that centrifugal force is "wrong." Though that may be a personal conviction, arguing such would be fruitless. Many competent scientists and engineers cling to the concept, and leading off with the assertion that they've been "wrong" all along seems a likely fast track to the trashcan or recycling bin. Beyond that, the issue of "right" and "wrong" in science is not as straightforward as typically taught in elementary school; it's actually deeply philosophical.

Engineering majors have been known to disparage philosophy majors. They boast that their work is grounded in science, but may fail to acknowledge that science is itself grounded in philosophy – unless they've gone on to pursue Ph.D.s with coursework in epistemology. The "Ph." in "Ph.D." stands for "philosophy," after all. Those who say it's merely "piled higher and deeper" would do well to consider that Ph.D.s are prominent in the aerospace enterprise. (I am not among them, being neither prominent nor Ph.D. My degree is Arch.D.)

A review of the rise of the Copernican over the Ptolemaic model of the cosmos is particularly relevant. Lakatos and Zahar [2] provide a good overview of various attempts to appraise the victory of Copernicus. Contrary to common presentation, Copernicus's model was neither simpler nor more accurate than Ptolemy's. They both confined celestial motions to perfect spheres, and tweaked or ignored observed data to suit their respective models. Kepler, Galileo, and Newton didn't merely build on Copernicus's model, but had to abandon certain elements of it to allow for a more physically dynamic force model. Ptolemy's model wasn't properly "falsified" until Bessel measured stellar parallax in 1838 – nearly 300 years after Copernicus published *On the Revolutions of the Celestial Spheres* (around 1543) and 150 years after Newton published his *Mathematical Principles of Natural Philosophy* (1687). (Newton himself referred to his work as principles of philosophy rather than laws of physics.) According to Lakatos and Zahar, Copernicus's approach was scientifically superior because it was less ad hoc and had more "predictive power," even if his model's actual predictions were not particularly superior (until after the model was further refined by Kepler).

Any "Copernican" or "Newtonian" system can by converted to a "Ptolemaic" system with equal accuracy simply by subtracting the predicted motion of the Earth from everything in the universe, including the Earth itself. Indeed, this is a necessary standard practice for much of Earth-based astronomy. But does this precede, or follow, a different fundamental understanding of the underlying physics? If I were to maintain that Newton's "Laws" are actually Newton's "Illusions," and that after applying them, there is a crucial final step to match reality by subtracting their prediction of Earth's motion, because of course the Earth doesn't *really* move … by what evidence would you prove me "wrong"? (That is not my view, but what if?)

I am not a "licensed philosopher" and AIAA is not primarily a philosophical society, so I will not pursue any

overarching definition of what distinguishes a "good" idea from a "bad" one. For the purposes of this paper, I define a "good" idea as one that makes a practical contribution to solving engineering problems, and a "bad" idea as one that introduces unnecessary complexity, contradiction, or mystery that impedes solutions.

In explaining orbit, weightlessness, and "artificial gravity," *centrifugal* is to *centripetal* as Ptolemy is to Newton. Judging from published designs for artificial-gravity habitats, the phantom forces conjured by adherents to the *centrifugal* concept *do not evidently* inform good architecture; moreover they *evidently do not* inform.

IV. Orbit: Centrifugal Force Contradicts Newton

The greatness of Newton's achievement is not only in its predictive power but also in its simplicity. The motions of celestial bodies, which wise men had mulled for millennia, were by Newton reduced to two simple formulas within the grasp of teenagers:

- **The Second Law of Motion**: The force required to accelerate a mass is directly proportional to both the mass and the acceleration:

$$\mathbf{F} = m \cdot \mathbf{A} \tag{1}$$

- **The Law of Gravitation**: The gravitational force between two masses is a mutually attractive force, directly proportional to a universal gravitational constant G and to each of the masses, and inversely proportional to the square of the distance between them:

$$F = G \cdot \frac{m_1 \cdot m_2}{\left| \mathbf{R}_1 - \mathbf{R}_2 \right|^2} \tag{2}$$

Because the mass terms in both laws represent the same physical property (inertial and gravitational mass are equivalent), we can set the equations equal, divide both sides by the equivalent mass, and find the acceleration of one mass toward another due to their mutual gravitational attraction:

$$
\begin{aligned}
A_1 &= F / m_1 = G \frac{m_2}{\left| \mathbf{R}_1 - \mathbf{R}_2 \right|^2} \\[2mm]
A_2 &= F / m_2 = G \frac{m_1}{\left| \mathbf{R}_1 - \mathbf{R}_2 \right|^2}
\end{aligned}
\tag{3}
$$

In Newtonian physics, which has served us very well for three centuries, these two laws comprise everything necessary to explain orbit (along with the observation that celestial bodies are roundish, not flat). Gravity is an attractive force: a dropped stone falls down toward the planet, not up or sideways. The gravitational force acting on an orbiting body, and its consequent acceleration, are unambiguously *centripetal* (directed *inward*) and explained in total by these two laws. Any supposed additional *centrifugal* (*outward*) force with non-zero magnitude would violate at least one of these laws. Since its magnitude must be zero or be in violation of highly reliable physics, why even mention it?

The essence of orbit is merely, while continuously falling, to be also moving fast enough "horizontally" to continuously overshoot the horizon of a roundish planet. Figure 1 illustrates this.

I consider it a failure of our elementary science education that so many people use centrifugal force to explain what holds a satellite up. Nothing holds a satellite up; gravity holds it down. Yet it's not hard to find on-line dis-educational material – not only text, but also well-produced influential videos – that insert centrifugal force into the discussion. (Try a web search for the keywords [orbit centrifugal].) Children hear such statements from trusted educators – or nowadays on the web – at an age when they're neither inclined nor prepared to challenge the consistency of the teaching.

> *"The circular motion of the satellite generates a centrifugal force."*

No. That begs the question of why the satellite is moving in a circular path to begin with. The circular motion itself manifests an unopposed, un-cancelled centripetal gravitational force. If anything cancelled that, it would not

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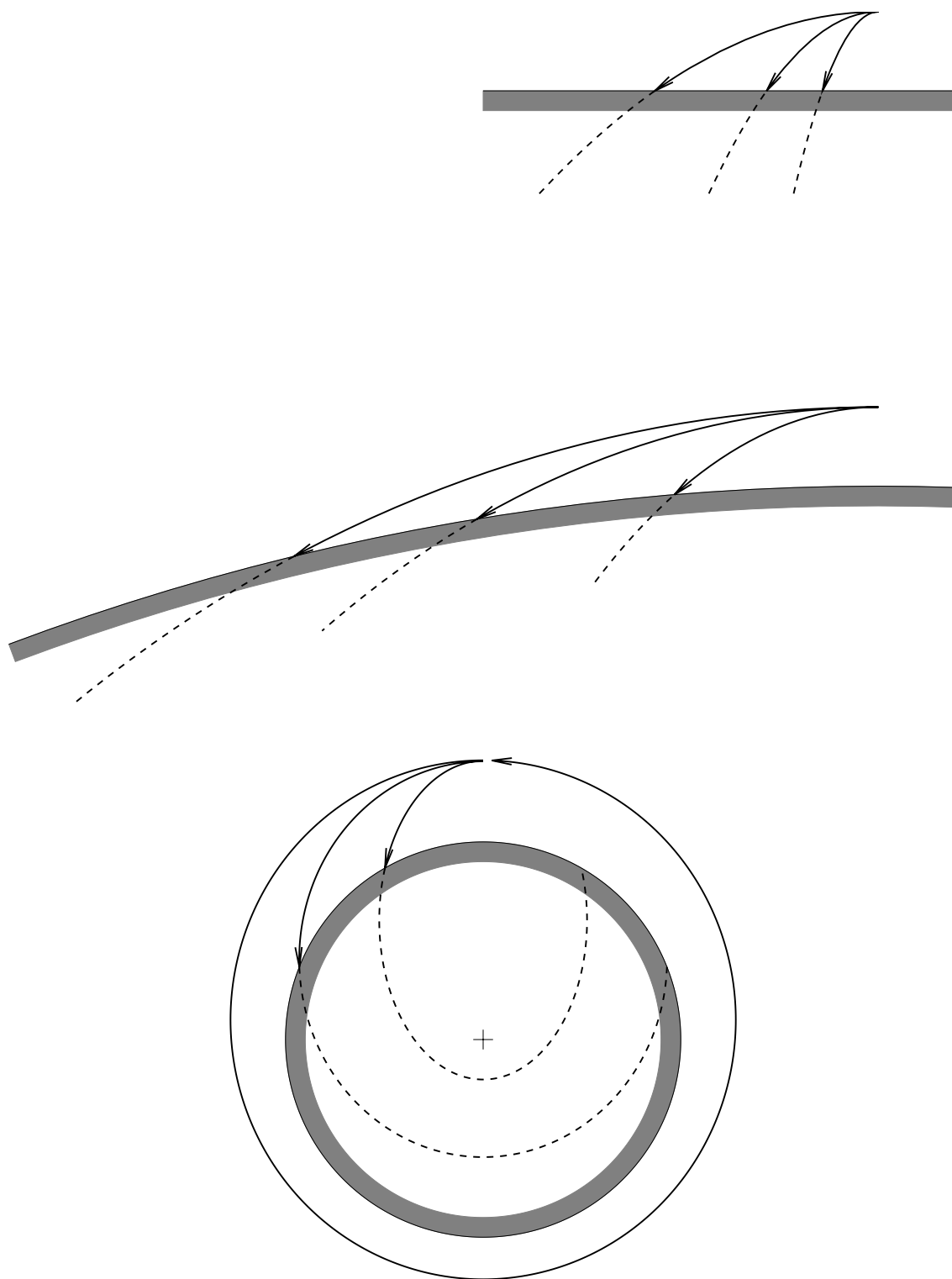
Fig. 1   Gravity alone, as a centripetal force, pulls a satellite down into orbit.   Sufficient horizontal velocity causes the satellite to continuously overshoot the horizon.

move in a circle. Moreover, the notion that circular motion "generates" a force conjures a completely inapplicable image of something like an electric generator. No such thing applies to orbit; there is no other force besides gravity that needs "generating."

> *"Newton's Third Law states that for every action there is an equal and opposite reaction, so there must be a centrifugal force reaction to the centripetal force."*

No. The equal and opposite force of the planet pulling on the satellite is the satellite pulling on the planet. The two bodies are subject to *equal and opposite centripetal* force and *centripetal* accelerations toward their mutual center of gravity. Given the Second Law of Motion and the ratio of the masses, we can predict that the effect on a big planet from a little satellite is immeasurably small – but not incalculably so.

V. Weightlessness: Centrifugal Force Cannot Account For It

Among the general population, there are many misconceptions for the cause of weightlessness in orbit. Some think that there's no gravity in space due to the vast distance from Earth. But, for example, the height of the International Space Station above Earth's surface is only about $1/20$ of Earth's radius, or $21/20$ times the radius from Earth's center. Consequently, according to Newton's Law of Gravitation (and some calculus that allows us to treat the mass of a sphere as if concentrated at its center), multiplying the denominator of Eq. (2) by $21/20$, the

intensity of Earth's gravitational field at the height of the International Space Station is about $(20/21)^2$ – about 90% – of the Earth's surface value. There's no shortage of gravity in orbit; else, there would be no orbit.

So, an ill-conceived argument goes, since there's plenty of gravity in orbit, weightlessness must be due to an opposing centrifugal force. But, as the previous section shows, there is no room for centrifugal force in the orbital mechanics of satellites, or astronauts.

Moreover, weightlessness does not depend on being in any circular orbital path. Launch an astronaut straight up in whatever reference frame you choose; launch toward the west instead of the east to cancel the Earth's rotation. Once he's above the effects of atmospheric drag, and the rocket engines cut off, he's weightless, despite a complete absence of circular motion, until he impacts the atmosphere again on his way back down. Centrifugal force cannot account for weightlessness in this case, so why invoke it in the orbital case?

This is not merely theoretical. Such straight-line near-Earth weightlessness is routinely exploited in Earth-based drop tubes in places such as the Zero Gravity Research Facility at the NASA Glenn Research Center [3]. These facilities do not merely simulate weightlessness. The few seconds of weightlessness that they provide is the same phenomenon as orbital weightlessness, until the experiment impacts the bottom of the tube.

## VI. Weight is Due to the Upward Push of the Floor (not the Downward Pull of Gravity)

Weightlessness is sometimes explained as "being in free-fall." That hints at an explanation, without quite achieving one. It leaves undefined what "free-fall" means. If we could put an astronaut in intergalactic space, as distant and balanced as possible among all gravitational influences, by what measure would he be falling? Would less falling lead to less weightlessness?

Contemporary physics posits four fundamental forces, or interactions, that account for all of the dynamics of the universe: Strong nuclear; Weak nuclear; Electromagnetic; and Gravitational. Among these, all but the Gravitational force conform to the Standard Model of quantum physics. Moreover, the Weak and Electromagnetic forces have been unified into an Electro-Weak force that was present immediately after the Big Bang, but soon bifurcated, as the universe cooled, into the separate forces apparent today. A long sought (but not yet found) Grand Unified Theory (GUT) would incorporate the Strong force as well, but still not include the Gravitational force. A theory that would finally unify all four fundamental forces would be a Theory of Everything (TOE). So far, gravity has resisted even quantization, let alone unification. Gravity is the odd man out. These concepts are covered in numerous books and papers. Davies [4] provides an authoritative overview, but Wikipedia also provides convenient articles on all of these concepts: "Fundamental Interaction", "Standard Model", "Electromagnetism", "Electroweak Interaction", "Grand Unified Theory", "Gravity", "General Relativity", and "Theory of Everything."

As Einstein developed his General Theory of Relativity, he viewed the phenomenon of gravity from a different perspective, which he described in a thought experiment: Put a man in a chest, out in deep space, far removed from any Newtonian gravitational source, and somehow accelerate the chest. Every experiment the man in the chest can
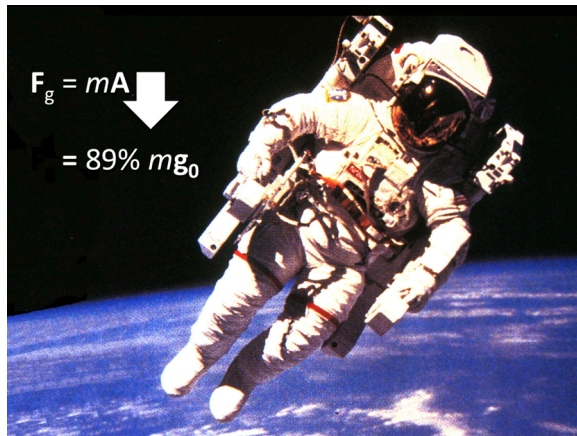
(a) Weightlessness
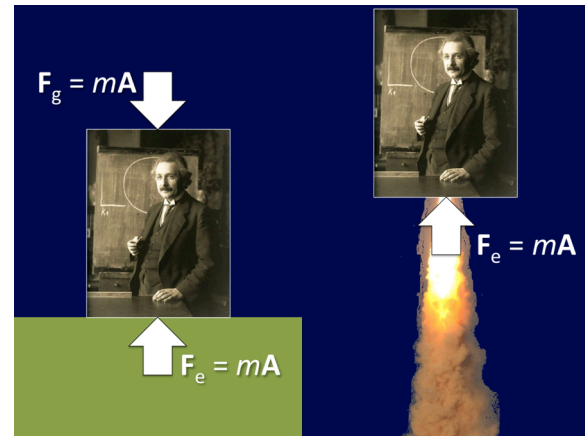
(b) Weight

Fig. 2 Gravitational force does not provide weight. Electromagnetic (mechanical) force does. Gravitational force is relevant only to the extent that it provokes an opposing electromagnetic force.

perform will run exactly as if the chest were suspended motionless in a gravitational field. For example, if he holds a stone in his hand, he will feel its weight; if he drops it, he will observe that it accelerates toward the floor. The pressure gradient in a fluid column, and the forces in rigid structures, will all conform to expectations in a gravitational field. Einstein concluded that, "a gravitational field exists for the man in the chest, despite the fact that there was no such field for the coordinate system first chosen" [5].

Einstein doesn't elaborate on what force accelerates the chest, other than to explicitly rule out Newtonian Gravity. But, the Strong and Weak forces operate only at the sub-atomic scale and are incapable of accelerating the chest. That leaves only the Electromagnetic force.

Moreover, all chemical and mechanical interactions – including *bio*chemical and *bio*mechanical – are due to the electromagnetic interaction between electron shells of adjacent atoms. All of the sensations and biomedical effects that we associate with weight (and weightlessness) are directly due to the electromagnetic interaction (or its absence), *not* due to *gravity*. This may seem counterintuitive, since we're taught from a very young age to associate weight with gravity. Gravity is relevant only to the extent that it pulls atoms together close enough for the electromagnetic force to operate. Figure 2 illustrates this.

On a planetary surface, it's useful to think of one's acceleration not merely as zero, but as the sum of two equal and opposite accelerations, due to different kinds of forces: gravitational down, and electromagnetic up. But, the gravitational force does not contribute to weight, as demonstrated by everyone who has ever experienced weightlessness: none of them has ever escaped gravity. What we have on a planetary surface, which astronauts in orbit do not have, is, essentially, an upward-accelerating floor. This is the essence of *not* being in free-fall: having a "floor" accelerating "up" – whether or not it's provoked or cancelled by a gravitational field. All of the forces we associate with weight conform to Newton's Second Law of Motion, $\mathbf{F} = m \cdot \mathbf{A}$, if we restrict the force to mechanical (electromagnetic, non-gravitational) interactions, and hold that there is always also a gravitational component to acceleration even if the net acceleration appears to be zero. Contact with the surface of a massive planet is one way, but not the only way, to provoke mechanical acceleration. Chemical rockets are another way. Rotating tensile structures that induce centripetal acceleration are another way.

(As a practical matter, it seems almost *as if* inertial space itself accelerates into mass, and mechanical contact with a planetary surface accelerates against that. But, that also may just be an illusion for someone unfamiliar with the geodesics of Einstein's four-dimensional spacetime.)

VII. Weight in a Rotating Space Habitat is Due to Centripetal Force

Perhaps part of the problem with *centrifugal* versus *centripetal* force is the way in which the formula is taught: a spoon-fed equation, without reference to its derivation, as if it were also a "law of physics" akin to Newton's Laws. But, it is not such a law; its genesis is quite different.

Newton's Laws describe relationships between different properties of material particles, based on observation,
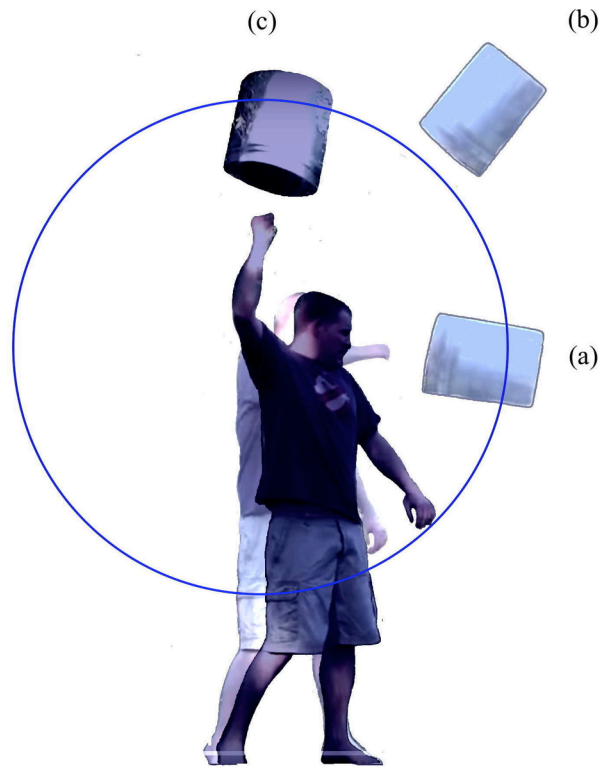
Fig. 3 With the upward tangential velocity at (a), momentum alone would have carried the water and bucket up to some point (b) on a vertical trajectory. Continuous centripetal acceleration pulls the bottom of the bucket inward against the water to deliver it to (c). [Image developed from frame grabs in Nick Lucid's Science Asylum video at https://www.youtube.com/ watch?v=Zjqrx7wrpJc, accessed on 2020-09-27.]

measurement, pattern recognition, and deduction. The Second Law of motion effectively *defines* the property of mass in terms of more intuitive phenomena of applied force and resultant acceleration.

In contrast, the formula for centripetal acceleration is a purely mathematical consequence of the very definitions of circular motion and acceleration, applying the principles of trigonometry and calculus, completely independent of any physical phenomenon. Whether or not Newton's Laws are true, centripetal acceleration is what it is, by definition. The Appendix of this paper outlines the derivation:

$$\mathbf{A}_{cent} = -\Omega^2 \cdot \mathbf{r} \tag{4}$$

where the minus sign indicates that the acceleration is inward, toward the center of rotation. Centripetal force is simply mass times centripetal acceleration, according to the Second Law of Motion.

The formula for *centrifugal* force is merely an arbitrary ± sign reversal of *centripetal* force to explain an illusion perceived from a "Ptolemaic" rotating frame of reference. (There is no centrifugal acceleration.)

Perhaps the association of weight with a *downward* pull of gravity leads some to conceive that "artificial gravity" must substitute an *outward centrifugal* force. But, as the previous sections argued, the perception of weight is due to the *upward* mechanical push of the floor. "Artificial gravity" achieves that with *inward centripetal* acceleration induced by structural tension in a rotating body. In retrospect, "artificial gravity" may be a misleading moniker, but it's with us to stay. As an analogy, we may speak of "sunrise" and "sunset" while understanding that they're actually illusions due to the spin of the Earth, not the orbit of the Sun. A suitable replacement phrase for "artificial gravity" is not readily apparent, but that should not prevent us from understanding what's really going on.

> *"If you swing a bucket of water over your head fast enough, centrifugal force will pin the water to the bottom of the bucket."*

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No. If you throw a bucket of water straight up, momentum carries it up beyond the point at which you release it. Nothing but its own momentum carries the water up to its apex. Consider the velocity of the water on the upswing, and where its momentum would have carried it, had you not provided centripetal acceleration with your arm. The water is pressed against the bottom of the bucket because that's where your arm and the bucket accelerated it to be. Moreover, the key word in "reaction" is "action" – "re-action." In the Newtonian frame of reference, at the apex of the swing, the only forces and the only actions on the bucket and the water are downward: gravitational and mechanical centripetal. Similar to the satellite tugging on the Earth, the re-action in this case is the equal and opposite centripetal force of the water and bucket on the person swinging it. This is especially apparent at the three o'clock and nine o'clock horizontal positions, where the person must shift their weight in the opposite direction to maintain balance, as shown in Fig. 3. When standing on a stable ground plane with adequate traction, that force is transferred to the planet. For a more dramatic isolation of the re-action on yourself, imagine swinging the same bucket of water while standing on roller skates.

VIII. The Centrifugal Force View Renders Coriolis Force Backward

Coriolis acceleration accompanies relative movement within a rotating frame of reference. Commonly conceived examples include moving around the circumference of a spinning torus, or "up" and "down" a ladder, but it applies to any relative motion, regardless of its orientation. The same mathematical process that yields the formula for centripetal acceleration also yields the formula for Coriolis acceleration. The Appendix of this paper outlines the derivation:

$$\mathbf{A}_{Cor} = 2 \cdot \mathbf{\Omega} \times \mathbf{v} \tag{5}$$

Coriolis force is mass times Coriolis acceleration.

As with the formula for centrifugal force, the formula for Coriolis force is often spoon-fed as if it were a fundamental law of physics, without reference to its purely mathematical genesis, with an arbitrary ± sign reversal to explain the illusory deviation of moving objects from expected motions, as if the habitat were not rotating – a Ptolemaic point of view [6, 7]. While this may be germane to studying the perceptions of rotating inhabitants, it is not very useful for solving design problems – especially in regard to the Coriolis forces that inhabitants will encounter.

Consider this scenario: An engineer is in an orbital space research facility (disconnected from the ground and any planetary influence on weight). He wants to measure the effects of centrifugal and Coriolis forces on a mouse. He instruments the mouse with sensors. The facility has a 10 m rotational radius. He intends to drop the mouse from a height of 2 m above the curved floor, but he doesn't want to damage it. Where should he position a pillow on the floor to safely catch the mouse? (Assume that it's a cordless USB mouse and the sensor data will be emitted over Bluetooth.)

As shown in Fig. 4(a), the rotating Ptolemaic view asserts that when he releases the mouse, centrifugal force pulls it toward the floor. But as soon as it starts to move, Coriolis force arises and pulls it toward the west, and the faster it moves, the faster it deflects. The solution appears to require a piecewise integration, probably with the aid of a computer, with time steps small enough to keep the solution within some finite tolerance. Perhaps you think the problem is under specified: I haven't stated the facility's rotation rate Ω or the "gravity level" at either the floor or 2 meters above it, so the centrifugal and Coriolis forces are unknown. The supposed centrifugal force is proportional to Ω^2. The Coriolis force is proportional to Ω, but also to the relative velocity v, which is increasing in magnitude as the mouse accelerates and also changing orientation relative to centrifugal force.

Now consider the situation from the Newtonian non-rotating frame of reference shown in Fig. 4(b). It becomes apparent that the solution is independent of rotation rate and gravity level and is completely determined by the geometry. The relative position of the mouse can be solved precisely for every point on the trajectory, with nothing more than high-school trigonometry and no necessity for piecewise integration or approximation (beyond the trig functions themselves). The mouse "falls" tangentially under no influence other than conservation of momentum, until its straight tangential path intersects the floor. It travels a distance $S = \sqrt{10^2 - 8^2} = 6$ m. Call that angle (for the falling "particle") $\theta_p = \arctan(6/8)$ radians. If it had not been dropped, it would have followed that same distance (6 m) in the same time around an arc with a radius of 8 m. Call that angle (for the rotating observer) $\theta_o = 6/8$ radians. That's the arc through which the engineer rotates during that time. The arc distance at the floor is

(a) Rotating frame of reference   (b) Inertial frame of reference
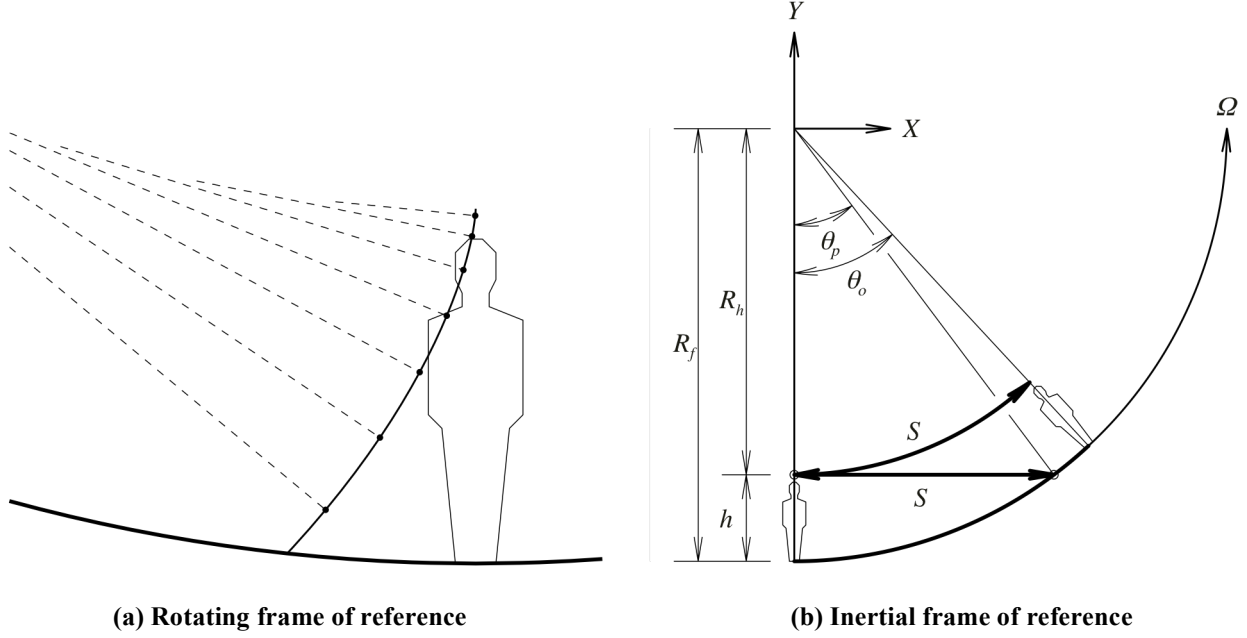
Fig. 4   Evaluating the path of a falling particle in artificial gravity: (a) using fictitious centrifugal and Coriolis forces in a rotating frame of reference; (b) using a Newtonian inertial frame of reference.

$\left(\theta_p - \theta_o\right)\cdot 10$ m.   It will fall to the west (negative angle, against the rotation) because $\forall n > 0 : \arctan(n) < n$. (Though, in general, if dropped from closer to the rotation axis, the angular difference could be less than $-\pi$).

The tangential path is like a thread unwinding from a spool, and the path the engineer sees is as if he were standing on the rim of the spinning spool; the falling mouse is at the endpoint of the unwinding thread, indicated by dashed lines in Fig. 4(a). It's always the same path, no matter how fast the spool spins or how fast the thread unwinds.

If the sensors on the mouse are functioning properly, they will record zero force (other than some slight aerodynamic drag). There is no Coriolis force that deflects the path of the falling mouse.

As mentioned previously, some authors prepend an arbitrary ± sign reversal on Eq. (5) to account for this illusion of deflection, induced by the rotation of the observer, but it contradicts the mathematical derivation. On the other hand, Eq. (5) would directly account for the illusory deflection if the definition of $\mathbf{\Omega}$ were reversed to represent the rotation of the universe relative to the stationary structure – consistent with a Ptolemaic point of view.

The mathematically derived Coriolis force describes the force that would be necessary to *prevent* the illusory deviation and constrain the mouse to a straight radial path in the rotating reference.

Consider the engineer climbing a ladder in the rotating space facility. As he ascends, he must lose tangential velocity, and as he descends he must gain velocity, as shown in Fig. 5(a). His tangential velocity at each point on the ladder is $\mathbf{\Omega} \times \mathbf{r}$. His relative velocity comprises a change in radius, $\mathbf{v} = \dot{\mathbf{r}}$, which entails a rate of change of tangential velocity – i.e., an acceleration – of $\mathbf{\Omega} \times \mathbf{v}$. That accounts for one factor of $\mathbf{\Omega} \times \mathbf{v}$. The formula for Coriolis acceleration has a multiplier of 2. The other contributor of $\mathbf{\Omega} \times \mathbf{v}$ is the continuous rotation of $\mathbf{v}$ relative to the inertial frame. Just as a rotating position vector entails a velocity of $\mathbf{\Omega} \times \mathbf{r}$, a rotating velocity vector entails an acceleration of $\mathbf{\Omega} \times \mathbf{v}$, whether or not $\mathbf{v}$ changes with respect to the rotating reference. So, the total Coriolis acceleration is $2 \cdot \mathbf{\Omega} \times \mathbf{v}$.

It is the rigid ladder that provides the necessary Coriolis force to accelerate the engineer as he climbs. Note that this force is in the opposite direction of the illusory deflection of free-falling particles. When ascending the ladder (toward the center), the force is toward the west, subtracting from his tangential velocity. When descending the ladder, the force is toward the east, adding to his tangential velocity. The total mechanical acceleration, centripetal plus Coriolis, comprises the climber's apparent "up" vector. The ladder has an apparent change in slope, relative to that up vector, that follows the form of a catenary arch [8]. It is critical to the engineer's safety that he stays on the

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| (a) Inertial frame of reference | (b) Climber's frame of reference |
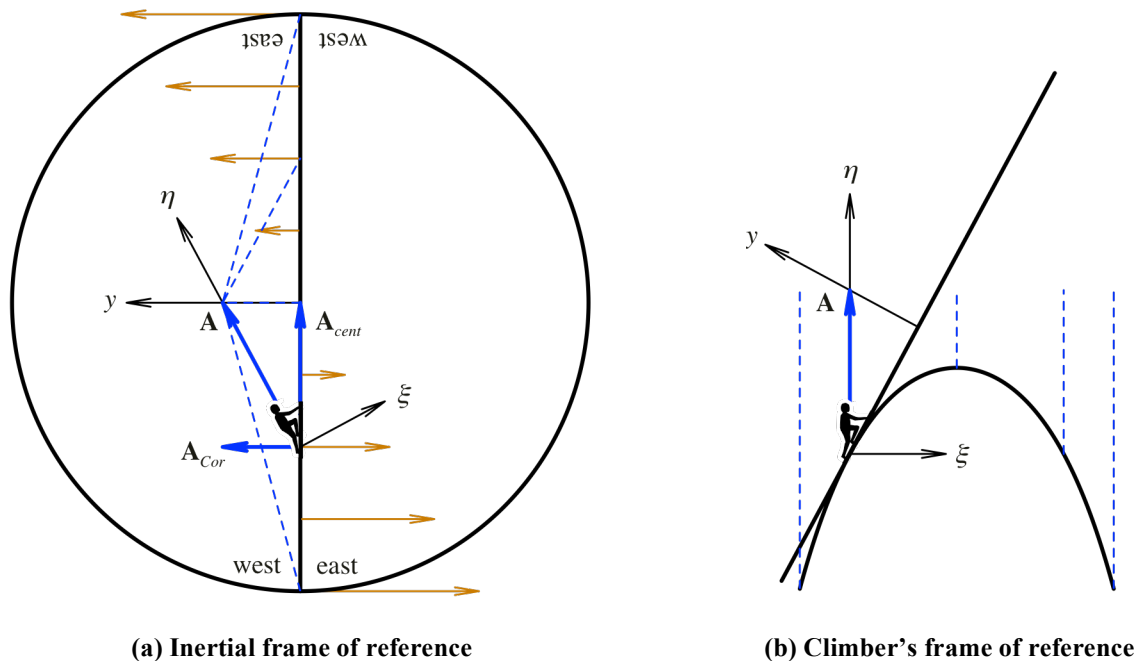
Fig. 5 Evaluating the actual centripetal and Coriolis forces acting on a person ascending a ladder in artificial-gravity: (a) in a Newtonian inertial frame of reference; (b) in the climber's apparent gravitational frame of reference, with an "up" vector aligned with the sum of the centripetal and Coriolis accelerations. The thin tan arrows in (a) indicate the tangential velocity. The thicker blue arrows indicate the centripetal, Coriolis, and total accelerations.

"top" side of the ladder: he should ascend on its west side and descend on its east side, as shown in Fig. 5(b). If he was wise, he designed and positioned the ladder to provide for that. The plane of the ladder should be perpendicular to the plane of rotation. Either it should be accessible from both sides with a double-sided floor cut, or separate ladders for ascending and descending should be provided on opposite sides of a single floor cut.

IX. Conclusion

Perhaps, misconception of the Newtonian forces acting on moving bodies in rotating habitats is a principal cause of conceptual flaws in designs for artificial gravity. In particular, accommodations for Coriolis force are absent from many such designs. Refs. [9, 10] discuss a few specific examples. Perhaps Coriolis force, like centrifugal force, is regarded as merely a "fictitious force," or perhaps it's regarded as too complex or non-intuitive to inject into design details. There may be an unexamined assumption that Coriolis force doesn't matter as long as the rotational parameters are kept within some limits of "comfort."

We can only guess as to why so many designs have evidently neglected to properly account for the Newtonian forces. Nevertheless, it seems likely that the "centrifugal force" concept has not helped: it is a concept that treats a rotating frame of reference as if it were not rotating. Neither Newton's Laws nor "common sense" apply to such accelerated frames of reference – except by acknowledging that the frame is indeed accelerated and evaluating the entire situation from a non-accelerated frame. Coriolis force is real and significant, but in the opposite sense to which it is described alongside centrifugal force. The level of comfort and even safety in a rotating habitat depends not only on the parameters of gravity level, spin rate, and radius, but also on the arrangement of the interior architecture with respect to the real centripetal and Coriolis forces.

The Ptolemaic system is non-optimal for understanding the motions of celestial bodies. In just the same way, "centrifugal force" is non-optimal for understanding artificial gravity, or for teaching anything about the physics of bodies in circular motion. It is better to use Newton's Laws to evaluate the motions of all of the bodies, and the forces acting on them, and then as a final step, subtract the motion of a rotating reference of interest to obtain relative coordinates.

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Appendix: The Mathematical Derivation of Centripetal and Coriolis Accelerations

This is a slightly revised edition of an appendix that I previously included in Ref [10], adapted to the context of this paper. I include it here also for convenience. This is inherently not unique or original research. This is a straightforward (though perhaps tedious) mathematical exercise. The essentials can be found in textbooks on mechanical dynamics.

The expressions for centripetal and Coriolis accelerations are not "laws of nature," but rather of pure mathematics. They derive from the very definitions of rotation and acceleration. The derivation relies on principles of trigonometry and vector calculus.

The position of a point (x, y, z) can be described as a vector from some selected center or origin (0, 0, 0):

$$\begin{aligned} \mathbf{r} &= r_x \cdot \mathbf{i} + r_y \cdot \mathbf{j} + r_z \cdot \mathbf{k} \\ &= x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k} \end{aligned} \tag{6}$$

where $\mathbf{r}$ is the position vector; $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit component vectors parallel to the x, y, and z axes; and r_x, r_y, and r_z are the projected lengths of $\mathbf{r}$ on those axes.

Velocity is the rate of change of position with time, denoted by a dot above the symbol:

$$\begin{aligned} \mathbf{v} = \dot{\mathbf{r}} &= \frac{d\mathbf{r}}{dt} \\ &= v_x \cdot \mathbf{i} + v_y \cdot \mathbf{j} + v_z \cdot \mathbf{k} \\ &= \dot{x} \cdot \mathbf{i} + \dot{y} \cdot \mathbf{j} + \dot{z} \cdot \mathbf{k} \end{aligned} \tag{7}$$

Acceleration is the rate of change of velocity:

$$\begin{aligned} \mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} &= \frac{d^2\mathbf{r}}{dt^2} \\ &= a_x \cdot \mathbf{i} + a_y \cdot \mathbf{j} + a_z \cdot \mathbf{k} \\ &= \ddot{x} \cdot \mathbf{i} + \ddot{y} \cdot \mathbf{j} + \ddot{z} \cdot \mathbf{k} \end{aligned} \tag{8}$$

If the xyz coordinate system is rotated around its z axis by an angle θ relative to an XYZ system that shares the same origin and $z = Z$ axis, then the coordinates of any point in the two systems have the following relationship:

$$\begin{aligned} X &= x \cdot \cos(\theta) - y \cdot \sin(\theta) \\ Y &= x \cdot \sin(\theta) + y \cdot \cos(\theta) \\ Z &= z \end{aligned} \tag{9}$$

To apply Newton's Second Law of Motion to evaluate the force acting on a particle in circular motion, we need to determine the acceleration of the particle in an inertial, non-accelerated (non-rotating) frame of reference. Let the xyz axes be a rotating frame of reference. (In the case of a rotating spacecraft, this is typically tied to the structure, but a physical structure is not necessary to the analysis.) Let the XYZ axes be a non-rotating inertial frame of reference with the same origin. Let the rotation of xyz relative to XYZ be around their shared $z = Z$ axis. The instantaneous rotation angle θ of the xyz reference is a function of the rotation rate Ω (in radians per unit time) and the elapsed time t:

$$\theta = \Omega \cdot t \tag{10}$$

Then the instantaneous position of a particle expressed in the rotating and inertial reference frames is:

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$$\mathbf{R} = \mathbf{r}$$
$$= x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k}$$
$$= X \cdot \mathbf{I} + Y \cdot \mathbf{J} + Z \cdot \mathbf{K} \tag{11}$$
$$= \left(x \cdot \cos(\Omega \cdot t) - y \cdot \sin(\Omega \cdot t)\right) \cdot \mathbf{I} + \left(x \cdot \sin(\Omega \cdot t) + y \cdot \cos(\Omega \cdot t)\right) \cdot \mathbf{J} + z \cdot \mathbf{K}$$

where $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{K}$ are unit component vectors parallel to the *X*, *Y*, and *Z* axes. The expressions for $\mathbf{R}$ and $\mathbf{r}$ represent the same instantaneous position, but in different frames of reference. This is somewhat analogous to a temperature being expressed as 32° F = 0° C, or a distance as 1 ft = 0.3048 m, except that here the **ijk** reference is continually changing relative to the **IJK** reference.

The first and second derivatives of Eq. (11) yield the inertial velocity and acceleration of the particle. These rely on a few results from elementary calculus: the derivatives of the sine and cosine functions

$$\frac{d}{dt} \sin(t) = \cos(t) \quad ; \quad \frac{d}{dt} \cos(t) = -\sin(t) \tag{12}$$

the chain rule for functions of functions

$$\frac{d}{dt} f\left(g(t)\right) = \frac{df}{dg} \cdot \frac{dg}{dt} \tag{13}$$

and the rule for products of functions

$$\frac{d}{dt}\left(f(t) \cdot g(t)\right) = f(t) \cdot \frac{dg}{dt} + \frac{df}{dt} \cdot g(t) \tag{14}$$

Applying the rules of Eqs. (12-14) to Eq. (11) yields the inertial velocity of the particle:

$$\mathbf{V} = \dot{\mathbf{R}}$$
$$= \left(-x \cdot \sin(\Omega \cdot t) \cdot \Omega + \dot{x} \cdot \cos(\Omega \cdot t) - y \cdot \cos(\Omega \cdot t) \cdot \Omega - \dot{y} \cdot \sin(\Omega \cdot t)\right) \cdot \mathbf{I}$$
$$+ \left(x \cdot \cos(\Omega \cdot t) \cdot \Omega + \dot{x} \cdot \sin(\Omega \cdot t) - y \cdot \sin(\Omega \cdot t) \cdot \Omega + \dot{y} \cdot \cos(\Omega \cdot t)\right) \cdot \mathbf{J}$$
$$+ \dot{z} \cdot \mathbf{K} \tag{15}$$
$$= -\Omega \cdot \left(x \cdot \sin(\Omega \cdot t) + y \cdot \cos(\Omega \cdot t)\right) \cdot \mathbf{I} + \Omega \cdot \left(x \cdot \cos(\Omega \cdot t) - y \cdot \sin(\Omega \cdot t)\right) \cdot \mathbf{J}$$
$$+ \left(\dot{x} \cdot \cos(\Omega \cdot t) - \dot{y} \cdot \sin(\Omega \cdot t)\right) \cdot \mathbf{I} + \left(\dot{x} \cdot \sin(\Omega \cdot t) + \dot{y} \cdot \cos(\Omega \cdot t)\right) \cdot \mathbf{J} + \dot{z} \cdot \mathbf{K}$$

There's a pattern in the plethora of terms in Eq. (15) that occurs so frequently that vector calculus provides an operator to encapsulate it: the cross-product. It has the form of a determinant of a 3×3 matrix comprising the basis vectors and the components of the vectors being crossed:

$$\mathbf{\Omega} \times \mathbf{r} = \begin{vmatrix} \mathbf{I} & \mathbf{J} & \mathbf{K} \\ \Omega_X & \Omega_Y & \Omega_Z \\ r_X & r_Y & r_Z \end{vmatrix} \tag{16}$$
$$= \left(\Omega_Y \cdot r_Z - \Omega_Z \cdot r_Y\right) \cdot \mathbf{I} + \left(\Omega_Z \cdot r_X - \Omega_X \cdot r_Z\right) \cdot \mathbf{J} + \left(\Omega_X \cdot r_Y - \Omega_Y \cdot r_X\right) \cdot \mathbf{K}$$

The cross-product of two vectors is a third vector that is perpendicular to both of the operands, with a magnitude (length) proportional to the sine of the angle between them. The cross-product of parallel vectors (in the same or opposite directions) is the zero vector $\mathbf{0}$. The $\mathbf{\Omega}$ vector is perpendicular to the plane of rotation (and to $\mathbf{r}$) and is oriented according to the right-hand rule. In this case, because the vectors are perpendicular, the sine is 1 and the magnitude of the cross-product is simply $\Omega \cdot r$.

Eq. (11) assumed a convenient coordinate system in which $\Omega_X = \Omega_Y = 0$ and $\Omega_Z = \Omega$, so several of the terms

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in the cross-product are zero. However, the pattern would apply for any arbitrary orientation of the rotation axis in the *XYZ* coordinate system. Eq. (15) can then be condensed to:

$$\mathbf{V} = \dot{\mathbf{R}} = \mathbf{\Omega} \times \mathbf{r} + \mathbf{v} \tag{17}$$

The first term on the right, $\mathbf{\Omega} \times \mathbf{r}$, is the tangential velocity of the particle in *XYZ* due to the rotation of the *xyz* reference. The second term, $\mathbf{v}$, is the velocity of the particle relative to the *xyz* reference.

The next-to-last line of Eq. (15) is the cross-product $\mathbf{\Omega} \times \mathbf{r}$ expressed in the *XYZ* reference (with **IJK** components). The last line of Eq. (15) expresses the instantaneous value of the relative velocity $\mathbf{v}$ transformed into the *XYZ* reference at time t – using Eq. (7), a rotation transform analogous to Eq. (9), and the instantaneous value of the rotation angle from Eq. (10).

The pattern in Eq. (17) is that the rate of change of the vector in the inertial *XYZ* system is equal to the cross-product of the angular velocity of the *xyz* system and the vector, plus the vector's relative rate of change in the rotating *xyz* system. This is known as the "Basic Kinematic Equation." It applies to any vector – not only for the change in position, but also for the change in velocity.

To find the inertial acceleration of the particle, we can apply the same pattern again:

$$\begin{aligned}
\mathbf{A} = \dot{\mathbf{V}} &= \ddot{\mathbf{R}} \\
&= \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r} + \mathbf{v}) + (\mathbf{\Omega} \times \mathbf{v} + \mathbf{a}) \\
&= \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) + 2 \cdot \mathbf{\Omega} \times \mathbf{v} + \mathbf{a} \\
&= -\Omega^2 \cdot \mathbf{r} + 2 \cdot \mathbf{\Omega} \times \mathbf{v} + \mathbf{a}
\end{aligned} \tag{18}$$

If one isn't comfortable with the shortcut taken from Eq. (17) to Eq. (18), one can reapply the rules of Eqs. (12-14) to all of the terms in Eq. (15), recombine, and ultimately arrive at the same result.

The first term on the right, $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$ or $-\Omega^2 \cdot \mathbf{r}$, is the centripetal acceleration. Each cross-product by $\mathbf{\Omega}$ rotates the result by $\pi/2$ radians. Two successive cross products reverse the direction. (This is evident by fully expanding the terms of the cross products.) The minus sign indicates that the acceleration is toward the center of rotation: centripetal, not centrifugal. The illusion of centrifugal force, caused by the rotation of the observer, omits the minus sign.

The second term, $2 \cdot \mathbf{\Omega} \times \mathbf{v}$, is the Coriolis acceleration. Note that there is no minus sign on the cross product. Some authors prepend a minus sign to try to explain illusions of force and acceleration perceived by a rotating observer – illusions caused by the acceleration of the observer himself.

The third term, $\mathbf{a}$, is the acceleration of the particle relative to the *xyz* reference.

In the case of circumferential motion on the curved "floor" of a cylinder or torus, either prograde or antigrade, $\mathbf{a}$ itself is another centripetal acceleration:

$$\begin{aligned}
\mathbf{a} &= \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{r}) \\
&= -\omega^2 \cdot \mathbf{r}
\end{aligned} \tag{19}$$

where $\mathbf{\omega}$ is the angular velocity relative to the rotating structure – e.g., walking speed divided by the radius of the arc: $\omega = v/r$. In this special case, we have:

$$\begin{aligned}
\mathbf{v} &= \mathbf{\omega} \times \mathbf{r} \\
\mathbf{V} &= \mathbf{\Omega} \times \mathbf{r} + \mathbf{v} \\
&= \mathbf{\Omega} \times \mathbf{r} + \mathbf{\omega} \times \mathbf{r} \\
&= (\mathbf{\Omega} + \mathbf{\omega}) \times \mathbf{r} \\
\mathbf{A} &= -\Omega^2 \cdot \mathbf{r} + 2 \cdot \mathbf{\Omega} \times \mathbf{v} + \mathbf{a} \\
&= -\Omega^2 \cdot \mathbf{r} + 2 \cdot \mathbf{\Omega} \times (\mathbf{\omega} \times \mathbf{r}) - \omega^2 \cdot \mathbf{r} \\
&= -(\Omega \pm \omega)^2 \cdot \mathbf{r}
\end{aligned} \tag{20}$$

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where the ± sign depends on whether $\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$ are in the same or opposite directions.

Many authors negate the centripetal and Coriolis terms in Eq. (18) to express the "fictitious" or "imaginary" centrifugal and Coriolis forces as seen in the rotating reference *as if* it were an inertial reference. This is analogous to describing planetary motions as Ptolemaic epicycles relative to a stationary Earth. While this may sometimes be useful, it is not conducive to understanding the operative physics. Newton's insights depended on a Copernican conception of Earth as an accelerated reference frame, and viewing it from an inertial reference beyond. The same is true for understanding artificial gravity in rotating structures.

To avoid subscripts, primes, hats, and other difficult-to-read diacritical marks (especially at small font sizes), this derivation has reserved lowercase symbols $(x, y, z, \mathbf{i}, \mathbf{j}, \mathbf{k}, \mathbf{r}, \mathbf{v}, \mathbf{a}, \boldsymbol{\omega})$ for measurements relative to the rotating coordinate system, and uppercase symbols $(X, Y, Z, \mathbf{I}, \mathbf{J}, \mathbf{K}, \mathbf{R}, \mathbf{V}, \mathbf{A}, \boldsymbol{\Omega})$ for measurements relative to the inertial coordinate system. The broader literature on physics and dynamics often reserves some of these symbols for other concepts not relevant to this discussion.

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