A Tether-Based Variable-Gravity Research Facility Concept

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The recent announcement of a return to the Moon and a mission to Mars has made the question of human response to lower levels of gravity more important. Recent advances in tether technology spurred by NASA's research in MXER tethers has led to a re-examination of the concept of a variable-gravity research facility (xGRF) for human research in low Earth orbit. Breakthroughs in simplified inertial tracking have made it possible to consider eliminating the "despun" section of previous designs. This, in turn, improves the prospect of a facility based entirely around a tether, with the human module on one end and a countermass on the other. With such a configuration, propellantless spinup and spindown is also possible based on the conservation of angular momentum from a gravity-gradient configuration to a spinning configuration. This not only saves large amounts of propellant but vastly simplifies crew and consumable resupply operations, since these can now be done in a microgravity configuration. The importance of the science to be obtained and the performance improvements in this new design argue strongly for further investigation.

Introduction

The desire to send humans to explore and settle the Moon and Mars is constrained by a very practical question, currently unanswered: what happens to the human body in gravity levels less than 1 g? The experience of long-duration exposure to microgravity, on space stations from Skylab to Mir to the International Space Station, has demonstrated that such exposure has undesirable effects on humans. Bone loss, immune system suppression, muscle atrophy, and a host of other medical conditions appear during such flights and make readaptation to Earth life more difficult. To counteract these problems, astronauts currently undertake a strenuous program of exercise while in space, to prepare their bodies for eventual return to the Earth. This exercise regimen has been shown to improve their recovery times after Earth return, but still leaves them typically in an immobilized condition upon landing, although they do recover over time.

The concept of using rotation to simulate artificial gravity has been proposed for many decades. Not only could a properly-rotating space station provide artificial gravity to its crew, but by varying the angular rate of the facility, it would be possible to simulate different levels of artificial gravity, thus allowing the crew to simulate the gravity levels of the Earth, Moon, or Mars, and test themselves and their equipment for operation in these environments.

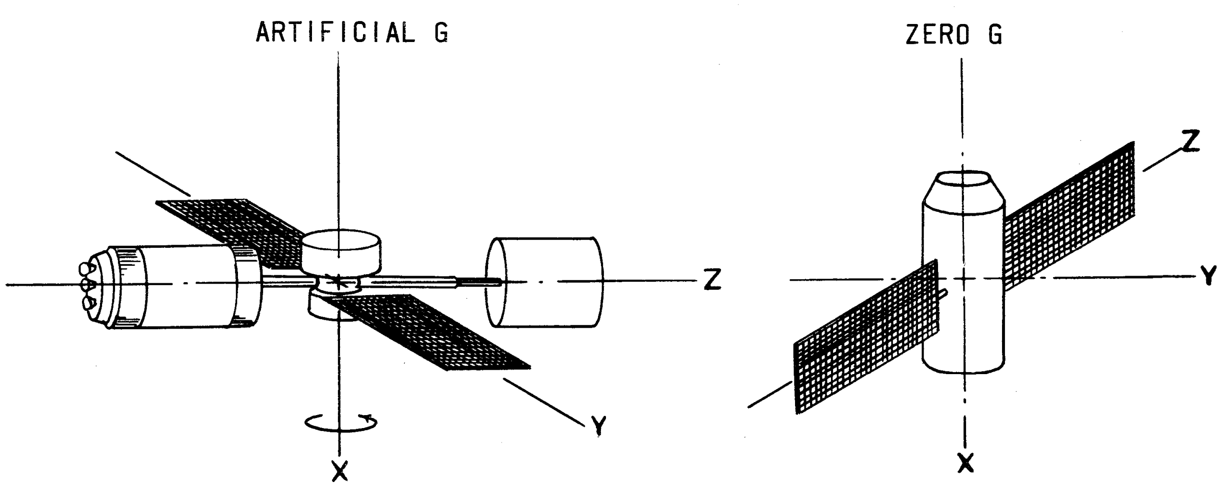


Figure 1: A comparison of artificial gravity and zero-g facilities from an early space station study (“Preliminary Technical Data for Earth Orbiting Space Station”, NASA MSC-EA-R-66-1, November 7, 1966.)

As space stations were first examined seriously in the 1960s, both “zero-g” and artificial gravity concepts were actively pursued, since planners were not sure whether humans could survive for extended periods without the presence of gravitational acceleration. In such trade studies, the operational challenges of the artificial gravity facilities were seen as greater than the “zero-g” facilities; they complicated basic operations such as solar array pointing, communications, rendezvous and docking, ingress and egress, and attitude control. This created a strong incentive for mission planners to assess whether a “zero-g” facility was adequate for their needs, and whether the human organism could be made to adapt to the environment rather than the other way around.

Human adaptation to microgravity has been fraught with difficulty and surprises. One of the early surprises was that humans seemed to fare quite well in the microgravity environment—after an early period of “space sickness” most astronauts became quite adapted to microgravity and even enjoyed it, as was shown on the American Skylab space station and the early Russian Salyut stations. But an insidious change was taking place in the astronauts’ bodies that threatened their return to Earth. Their bones, unloaded from the stresses of operating under gravitational loads, were leaching calcium from the bone structure into the blood, where it was excreted. This process of decalcification appeared to continue throughout the duration of the space mission, and left the astronauts dangerously weak when they returned to Earth, much like elderly persons who suffer from osteoporosis.

Figure 2: Long-duration space flight leaves cosmonauts weak and unable to stand on their own.

It was clear that the load-bearing bones need to experience stress during the space mission in order to slow the loss of calcium. Strenuous exercise was proposed as a remediation measure during long space flights, and Russian experiences on the Salyut 6 space station in the late 1970s showed that 2-3 hours a day of strenuous exercise could significantly reduce the effects of decalcification on bone structure. Indeed, it has now become common practice on later space stations, such as Salyut 7, Mir and the International Space Station, for crews to engage in strenuous exercise each day to combat the effects of microgravity.

But this countermeasure has a limited effectiveness and comes at a great cost. The exercise regimes have not been shown to stop or reverse the effects on bone structure, only to slow them, and there are other effects to the body from prolonged microgravity exposure, including immune system suppression and alternations in blood flow patterns that are not counteracted by the exercise regime. The great cost of the exercise is in useful crew time, a significant fraction of which is spent in preparation, execution, and cleanup from the exercise regime. As we ponder missions to the Moon and Mars which may last months and years, the cumulative loss of valuable crew time is a significant concern.

Consider a crew on the surface of Mars. Were they to engage in the same type of exercise countermeasures used on the International Space Station, they would consume about 2-3 hours per day of waking time in activities related to exercise. If we assume that they are awake 16 hours per day and that roughly 8 of these hours are engaged in meal preparation, hygiene, and crew personal time, we see that only leaves about 5-6 hours per day for scientific observations and extravehicular activities. The “cost” of all this exercise time is not insignificant.

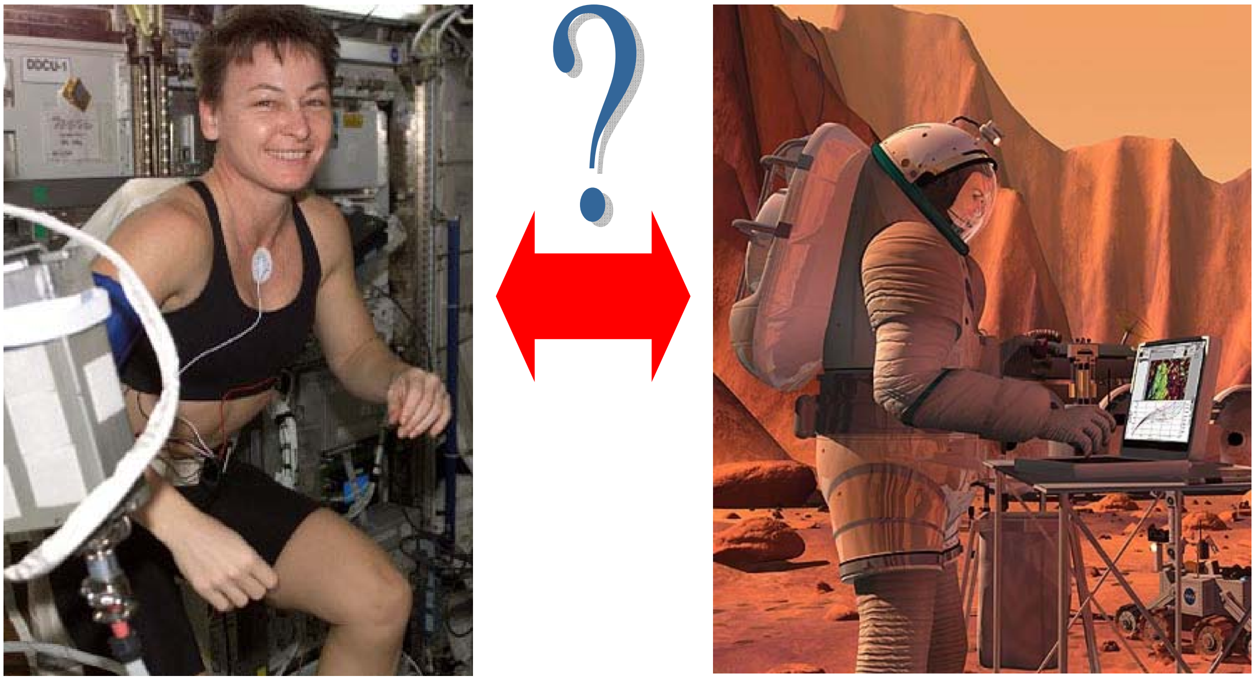


Figure 3: Knowing the human response to low gravity could mean the difference between exploring or exercising!

But on Mars, with its gravity 3/8 that of Earth's, is it necessary to engage in such a level of exercise? Is the gravitational environment sufficient to produce the needed stresses and eliminate the need for exercise as a countermeasure? And if so, how could we know before we go? Where could we simulate a gravity level similar to Mars and assess human performance? Unfortunately, it is not possible to simulate such a gravitational environment for extended periods of time on the Earth. Brief intervals could be simulated by high-altitude aircraft flying near-parabolic trajectories, and inclined bedrest could simulate (poorly) the loading patterns on the body, but there is no way to simulate Mars gravity for months at a time in a way that specimens could move about and engage in normal activities.

In Earth orbit, in a rotating facility, such conditions could be accurately simulated for extended periods of time. The cost of doing so would be significantly less than sending crews to Mars and "finding out" what happens to them in that environment. Indeed, we would never threaten the health of a crew on Mars by asking them to refrain from exercise countermeasures to use them as a "control group" to assess response versus a group that exercises. But without such scientific investigation, we will be forced to resort to the application of microgravity countermeasures, neither knowing if they are too much or too little for the "mini-gravity" environment of the Moon and Mars. Furthermore, essential engineering data on the performance of equipment and life-support systems in lower levels of gravity could be obtained.

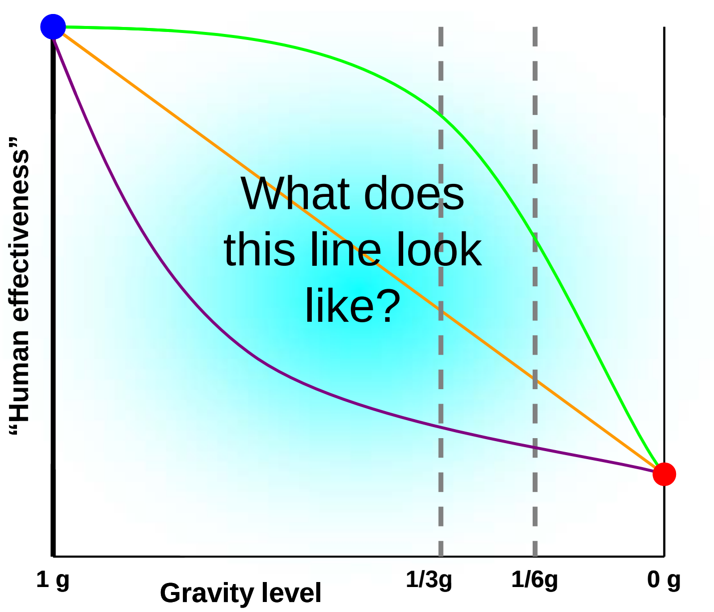


Figure 4: At the risk of oversimplification, we know that Earth gravity is fine and zero gravity is not, but what about in-between?

Previous Design Concepts and Issues for Variable-Gravity Research Facilities

In a sense, we return to the "fork-in-the-road" taken in the mid-1960s, when microgravity and artificial gravity facilities were evaluated one against another. The microgravity space station path has been taken to its logical conclusion for the past 30 years and found wanting—our countermeasures to long-term human microgravity exposure are insufficient to insure the long-term well-being of the crew. Thus let us examine the other, "complicated" option, artificial gravity, and see if we can get it to work acceptably.

Inherent in any design for an artificial-gravity facility is the need for a facility to rotate sufficiently to produce the desired acceleration. The acceleration (a) is a function of both the moment arm (r, the distance from the rotational center) and the angular rate (ω).

$$a = \omega^2 r \tag{1}$$

With these two variables, one can be traded off against another to achieve the desired acceleration. Obviously, it would be attractive to increase angular rate to decrease the rotational moment arm, but there are real constraints on this—angular rate cannot be made so high that the crew feels significant discomfort or sickness. Several studies have attempted to quantify the maximum angular rate that can be tolerated by humans over an extended period of time. The results have been inconclusive and have ranged from 2-6 RPM, mainly due to the difficulty of accurately measuring human response to the angular rate.

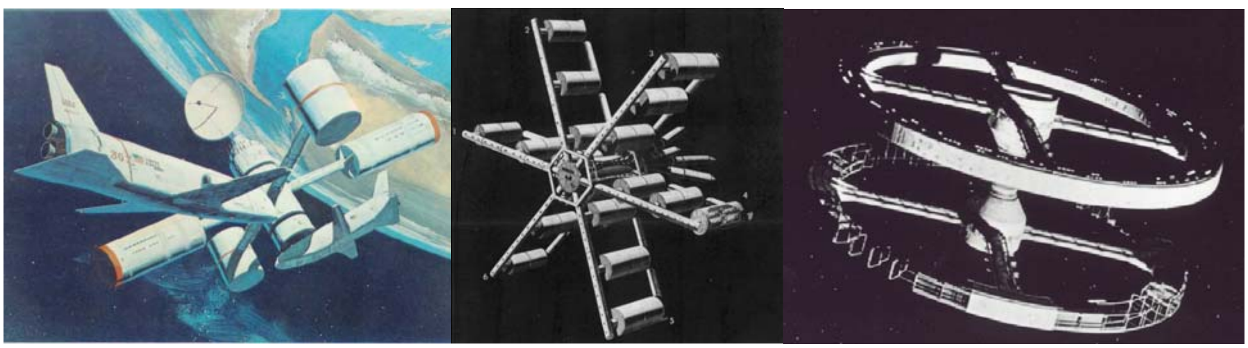


Figure 5: A variety of artificial-gravity space stations concepts, all with constant moment-of-inertia configurations.

But given the maximum angular rate that can be tolerated and the design level of acceleration, the basic length of the moment arm can be quantified, and that fundamental length constitutes a sort of "yardstick" in the design of artificial gravity facilities. We see in Figure 5 a number of artificial gravity station concepts, each of which is fundamentally sized by the length of the rotational moment arm. Each of these is also an example of a configuration with a fixed moment-of-inertia, which is a basic consideration in determining the spin-up and spin-down of such a facility.

The design of the facility is further complicated by all the "irreducible complexity" of a manned space station. Obviously, there must be stability, power, energy storage, and crew exchange capability. There must be the capability to communicate with the ground and/or data relay satellites. All of these tasks are made much more difficult, at least on first examination, by a space station that is rotating.

Design RPM	*Moment arm (for 1.0 g)*
1.0	895 m
2.0	224 m
3.0	99 m
4.0	56 m
5.0	36 m
6.0	25 m

Table 1: The length of the moment arm for a given artificial gravity level and angular rate represents a "yardstick" in the size of a rotating facility.

Let us consider power, communications, and attitude control. All of these have the common element of requiring some type of pointing. The power system (ostensibly a solar array) must point toward the Sun. The communications antenna must point toward the ground or to a data relay satellite. The attitude control thrusters must point in some inertial orientation to create the necessary torques and translations to stabilize the facility. On a rotating space station, all of these pointing directions are in constant movement relative to the facility itself and require agile tracking systems.

Then there is the issue of getting crew on and off the station. If, as it will be shown, there is a substantial penalty for spinning up and spinning down the facility using thrusters, then the alternative is to direct the crew and supply transfer to a location on the facility that is either not rotating or rotating very slowly. Thus we see the common feature of a "despun" or slowly-spinning hub on many of these rotating station concepts. But the hub must then be connected to the pressurized modules that are under artificial gravity—either by a long pressurized tunnel or by some sort of elevator car. Either option is more complex than simply docking the crew and supply craft directly to the pressurized module—but that requires spin-up and spin-down.

There is also the significant question of crew escape. If the crew for some reason needs to leave the facility and return to Earth, on very short notice, how do they do so? Is there an emergency-return vehicle? Where is it located—at the hub? Do they have to "climb the ladder" or ride an elevator to get there? What if one of them is severely injured and requires immediate medical attention? Do they despin the station, and how long will that take?

So this is a classic case of "picking your pain" in engineering—which design option entails less cost and risk? Until very recently, it would have been the "classic" design that has been seen in the previous pictures. But very recent developments in tether technology, energy storage, tracking systems, and manned vehicles make a new and much simpler option finally feasible.

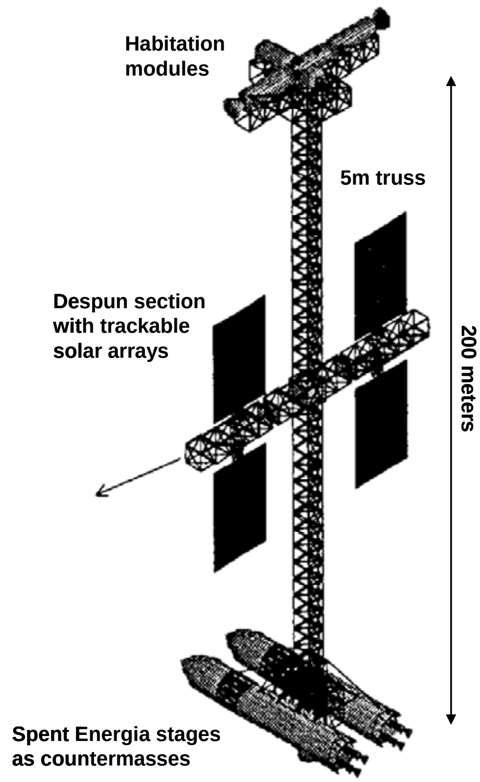
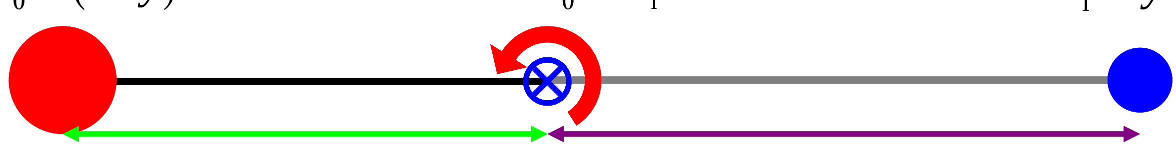


Figure 6: An artificial-gravity facility concept developed at the 1989 International Space University summer session.

Analysis of a Simplified Facility

In the interest of analyzing some different options, let us examine a facility that is essentially a dumbbell, with a manned module on one end and a countermass on the other. Let us further assume that the two masses are connected by a massless structure such as a truss or tether. Given the design moment arm (r_0) and the total mass (M) of the facility (manned module mass plus countermass), it is useful to define a countermass fraction (y)—a fraction of the total mass that comprises the countermass. With this definition, the total separation length (ℓ) between the manned module and the countermass can be found as a function of the countermass fraction and the design moment arm alone.

$$m_0 = (1-y)M \qquad M = m_0 + m_1 \qquad m_1 = yM$$



$$r_0 = \frac{a_1}{\omega_1^2} = \frac{m_1 \ell}{M} = \frac{yM\ell}{M} = y\ell \qquad r_1 = \frac{m_0 \ell}{M} = \frac{(1-y)M\ell}{M} = (1-y)\ell$$

Figure 7: A "dumbbell" model of the facility, connected by a massless structure, showing the relationships between the total mass, pressurized module mass, moment arm lengths, and total length.

The basic question before us is how should the facility spin up to the desired spin rate? Based on the basic equation for angular momentum:

$$H = I\omega \tag{2}$$

It is clear that if a given value of ω (angular rate) is the goal, then there are two ways to achieve it. One is to alter the total angular momentum of the system, leaving the moment-of-inertia fixed, until the desired angular rate is achieved. The other assumes that the initial angular momentum is greater than zero, and proceeds to alter the moment-of-inertia until the desired angular rate is achieved. In simpler terms, the first technique uses rockets to spin up the facility and change the angular momentum, whereas the second retracts tether to change the moment-of-inertia and increase the spin rate, much like a figure skater pulling her arms in during a spin.

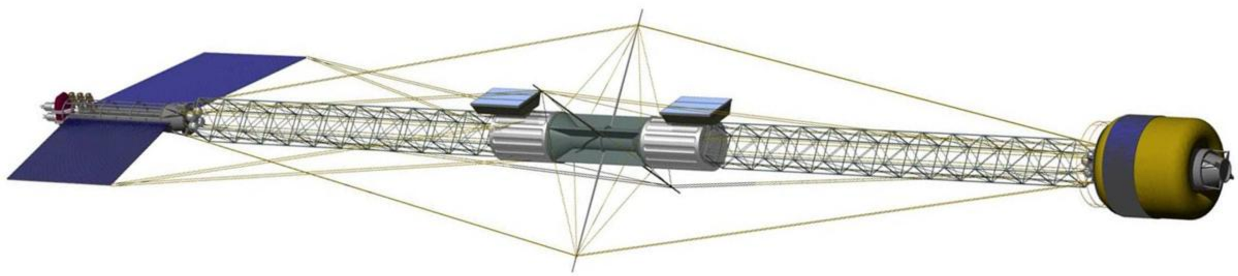


Figure 8: A recent concept for a Mars exploration vehicle using nuclear electric propulsion and artificial gravity. It has a pressurized module on one end and the nuclear reactor, power conversion system, and radiators on the other. It also has a fixed moment-of-inertia and relies on propulsive spinup to achieve the angular rate necessary to create the desired artificial gravity at the pressurized module.

Constant Moment-of-Inertia, Variable Angular Momentum (propulsive maneuvers to change angular rate)

Let us first examine the case of the fixed moment-of-inertia and the propulsive spin-up. This technique has the advantage that it does not require any initial rotation rate to begin. But what is the proper length of the tether and should the countermass be large or small? Before examining the equations, simple reasoning says that if the countermass is small, the separation length will be large, and so will the moment-of-inertia. But if the spin-up thrusters are located on the countermass, then they will have a long moment arm with which to change the angular momentum. So which effect will dominate? Let us solve for the moment-of-inertia of the system in terms of the pressurized module mass (m_0), the design moment arm (r_0), and the countermass fraction (y).

$$I = m_0 r_0^{\,2} + m_1 r_1^{\,2} = m_0 r_0^{\,2} + m_0\left(\frac{y}{1-y}\right) r_0^{\,2}\left(\frac{1-y}{y}\right)^2 = \frac{m_0 r_0^{\,2}}{y} \tag{3}$$

As expected, the moment-of-inertia is inversely proportional to countermass fraction, so as the countermass gets smaller (which is desirable) the separation length increases and the moment-of-inertia increases. Now let us define the angular impulse (J) as the integral of propulsive force over time multiplied by the length of the moment arm. The angular impulse is also the change in angular momentum and, for a facility with a constant moment-of-inertia, proportional to the change in angular rate.

$$J = r\int F dt = I\Delta\omega = \Delta H \tag{4}$$

If we assume that the spin thrusters are mounted on the countermass so as to maximize their effectiveness, we can substitute the definition of the countermass moment arm (r_1) into the equation and simplify.

$$r_1 = r_0\left(\frac{1-y}{y}\right) \tag{5}$$

$$\int F dt = \frac{J}{r_1} = \frac{I\Delta\omega}{r_1} = \frac{m_0 r_0^{\ 2}\Delta\omega}{y}\left(\frac{y}{r_0(1-y)}\right) = \frac{m_0 r_0 \Delta\omega}{(1-y)} \tag{6}$$

If we further assume that we begin with an angular rate of zero, then the design angular rate (ω) is essentially the same as the change in angular rate (Δω); this allows further simplifications in the equation by substituting the definition of the design moment arm ($r_0 = a/\omega^2$).

$$\int F dt = \frac{m_0 r_0 \Delta\omega}{(1-y)} \approx \frac{m_0 \Delta\omega}{(1-y)}\left(\frac{a}{\omega^2}\right) = \frac{m_0 a}{\omega(1-y)} \tag{7}$$

Therefore, if we assume that the mass of the pressurized module, the desired acceleration, and the design angular rate are given, the only variable we can use to minimize the propulsive impulse required to spin-up the facility is the countermass fraction, which this equation indicates should be minimized. This is a somewhat surprising result, as it might have been expected that the moment-of-inertia, which was quadratic with length, would dominate over the spin-up torque, which was only linear with length. Nevertheless, any facility that uses spin-up thrusters must have a "smart" countermass and deal with the issues of resupply of propellants to the countermass and active control.

Constant Angular Momentum, Variable Moment-of-Inertia (tether reeling to change angular rate)

Let us examine the other case, altering moment-of-inertia and conserving angular momentum to achieve the desired spin rate. Let us again return to the equation for moment-of-inertia, this time solving it in terms of total mass and total separation distance.

$$I = m_0 r_0^{\ 2} + m_1 r_1^{\ 2} = (1-y)My^2\ell^2 + yM(1-y)^2\ell^2 = M\ell^2 y(1-y)(y+1-y) \tag{8}$$

$$I = M\ell^2 y(1-y) \tag{9}$$

This form of the moment-of-inertia equation can be shown to be identical to the previous derivation, in the limiting case of a separation distance required for the design moment arm and countermass fraction. But the real utility of this new form of the equation is in cases where the separation distance is varied to achieve a variable moment-of-inertia. This equation also shows the quadratic dependence of moment-of-inertia on total tether length. If we assume that the facility is initially oriented along the radial direction in its orbit around the Earth (gravity-gradient configuration) then it will have some rotation rate commensurate with that orbital period, and hence, some initial value of angular momentum. Let us assume that the subscript 0 refers to the fully-extended length and the subscript 1 refers to the length in the retracted, design condition.

$$H = M\ell_0^{\ 2} y(1-y)\omega_0 = M\ell_1^{\ 2} y(1-y)\omega_1 \tag{10}$$

Assuming that angular momentum is conserved during tether reeling (which can be achieved through proper timing of tether extension or retraction) then the ratio between initial angular rate and final angular rate can be shown to be a ratio of initial and final length. The subscripts 0 and 1 again correspond to the extended and retracted configurations.

$$\frac{\omega_1}{\omega_0} = \frac{M\ell_0^{\ 2} y(1-y)}{M\ell_1^{\ 2} y(1-y)} = \left(\frac{\ell_0}{\ell_1}\right)^2 \tag{11}$$

Hence, for a desired angular rate, this equation can be used to assess the amount of tether that must be retracted to achieve that rate for a given initial angular rate. The angular rate ratio will then be the square of the retraction ratio (initial length/retracted length). Thus, within these simplifying assumptions, the spinup of a rotating facility is independent of its total mass and its mass distribution, but only depends on the retraction ratio.

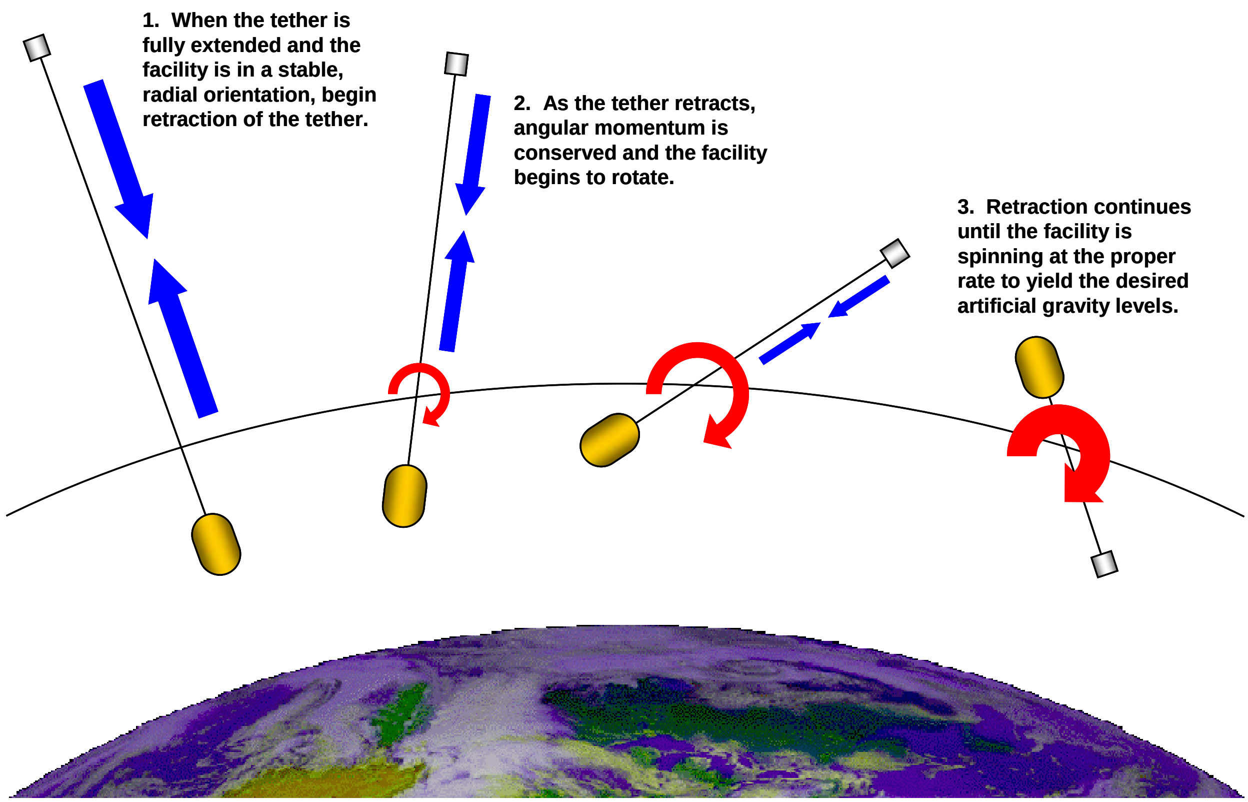


Figure 9: The facility can be spun-up through a tether retraction technique.

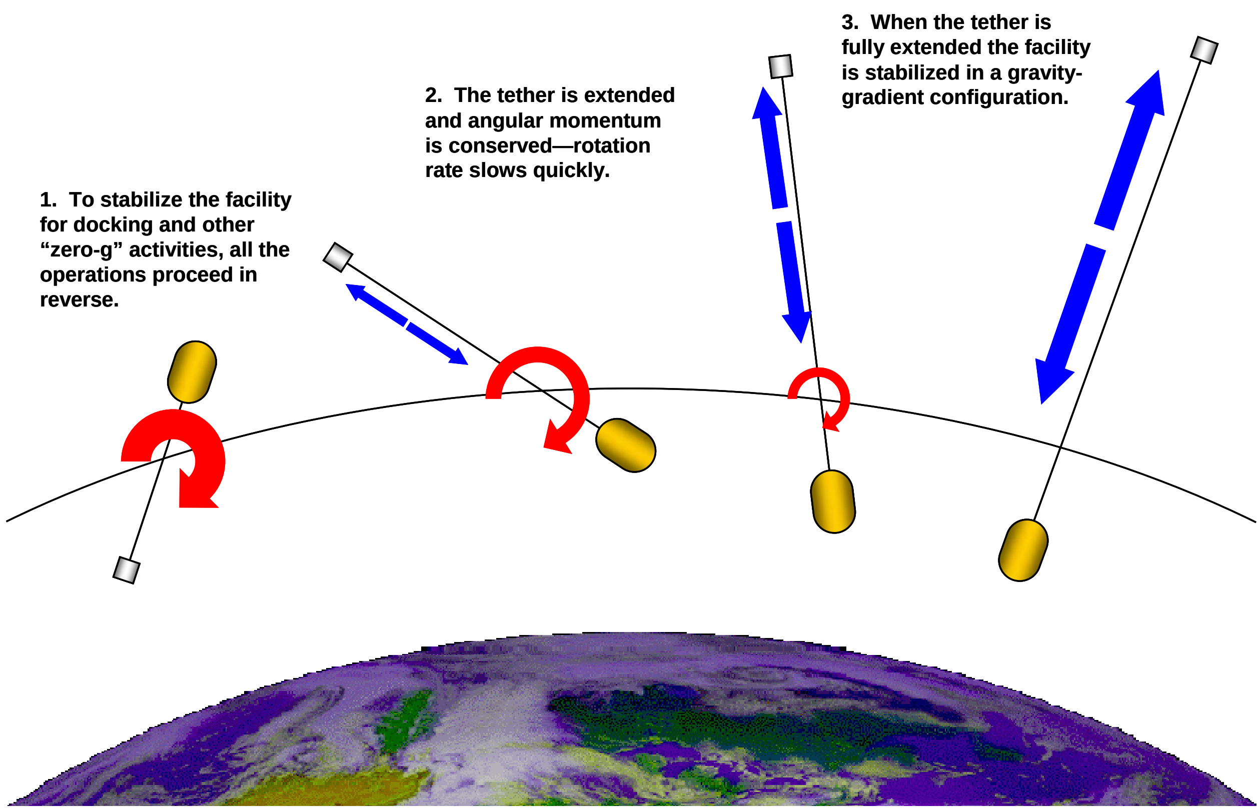


Figure 10: The facility can be de-spun in a similar yet reverse manner.

The significance of this equation is in its independence on total mass and mass distribution. This means that no matter what the total mass of the facility, no matter what the countermass fraction, no matter what the final length, this spinup technique could be used. This gives the designer an enormous amount of additional flexibility in the design of the facility, since now all of the "smart" mass (pressurized module, power system, tether reel and deployer) can be located at one end of the facility and the countermass can be "dumb" mass, such as the spent upper stage of the rocket that launched the facility initially.

Design Concept and Concept of Operations of the simplified xGRF

It is clear that the resources to pursue a variable-gravity research facility in space will not be available unless the costs of such a facility are reduced dramatically. And in order to reduce costs, the mass and complexity of the facility must be reduced dramatically. A tentative goal for this study was to reduce the mass of the xGRF to the level where the facility might be launched on a single flight of a large rocket such as the Delta IV Heavy, which can carry a 24 metric tonne payload to low Earth orbit.

Of course, it is not sufficient to merely hope for cost and mass reductions—there must be real design innovations that "change the equation", so to speak, and permit a design solution that is actually simpler and cheaper. The main advantages to pursue in this design for mass and cost reduction are

- Simplified spin control through tether reeling (variable moment-of-inertia)
- The elimination of any despun elements through the spin control technique and the use of unique pointing mechanisms (Canfield joints).
- Simplification of crew exchange, consumable resupply, and emergency escape, through the use of standard "zero-g" docking made possible through the ease of spinup and spindown.

The design concept for consideration is a facility that is much like a simple dumbbell connected by a variable-length tether. On one end of the facility would be the pressurized module where the crew would live and conduct their experiments. For mass savings, this module would probably be an inflatable pressurized module based on the "Transhab" technologies developed at Johnson Space Center in the late 1990s and now being pursued by Bigelow Aerospace. At the one end of the pressurized module would be a docking port for an unmanned logistics module or a manned capsule such as the proposed Crew Exploration Vehicle. The docking port would be designed for low-gravity operations but would include "hard-docking" mechanisms that could firmly attach the crew transfer vehicle to the facility while it is spinning (during artificial-gravity operations). This continuous connection to the facility allows the crew transfer vehicle to serve as an "escape module" for the crew in the event that they needed to leave the xGRF and return to Earth rapidly.

In order to pursue this simple mode of rendezvous and docking operations, it is necessary to have a mechanism for spinning up and spinning down the facility rather quickly. A spinup/spindown technique based on tether reeling has been proposed which accomplishes this need without the consumption of propellant. Furthermore, this technique can reduce spin rate very rapidly in the event an evacuation is required as well as generate emergency power, if needed, during such an evacuation. While angular momentum is conserved, rotational energy is decreased, and electrical power would be generated by the reeling motor as the tether is reeled out. To reel the tether in, rotational energy must be added, and this energy (to drive the reel) would be provided by flywheel energy storage systems which would be charged by solar panels.

Another problem with such a simple facility that has been raised in past studies is the difficulty of tracking inertial and pseudo-inertial targets from a rotating facility. To generate power, solar arrays are required to track the Sun. Communications probably must track a pseudo-inertial target such a TDRS satellite in geosynchronous orbit. And thrusters would need to be able to point at inertial thrust vectors during rotating operation.

Figure 11: xGRF concept

Solar arrays, communications systems, and thrusters would all need to transfer power, data, and fluids across rotating joints or slip rings if they were to be mounted near the pressurized module and yet fulfill their duties. These fluid, data, and power transfers could become very unpleasant development problems. Fortunately, several recent innovations make such operation possible without any sort of rotating connections.

The key is a unique mechanical joint developed by Dr. Stephen Canfield of Tennessee Technological University. The "Trio Tri-Star Carpal Wrist Robotic Joint", mercifully referred to simply as the Canfield joint, is a "parallel" mechanism that allows a structure to point at an inertial target while being mounted to a rotating platform. Fluid, power, or data connections can be made from one side of the joint to the other because the two sides of the joint do not differentially rotate, even though the joint itself may be bent 90 degrees or more.

The kinematic behavior of the Canfield joint is extremely difficult to describe in words, and not much easier to discern in still pictures, but suffice it to say, seeing is believing—the joint is truly capable of smooth pointing and orientation within a hemisphere.

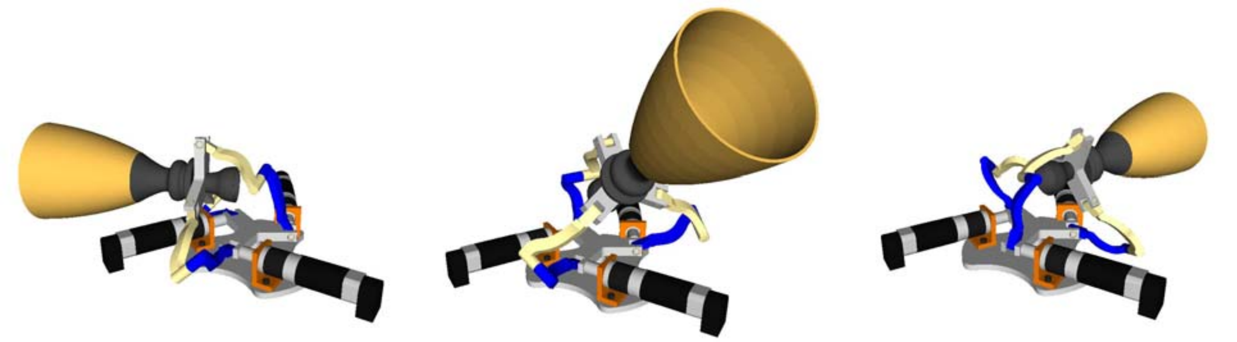


Figure 12: A series of still pictures showing a typical reaction-control thruster mounted on a Canfield joint, capable of pointing any direction within a hemisphere.

The parallel structure of the joint not only gives it tremendous strength, but three degrees of freedom—altitude, azimuth, and "plunge". The first two degrees are sufficient to allow it to point to any direction within a hemispheric workplane. And unlike other structures, the joint can access this entire workspace without encountering mathematical "singularities" that make solution of the base angles impossible. The joint is driven by three actuators on each of the three base legs. Any commanded direction corresponds to a unique solution of the three angles between the base and the base legs.

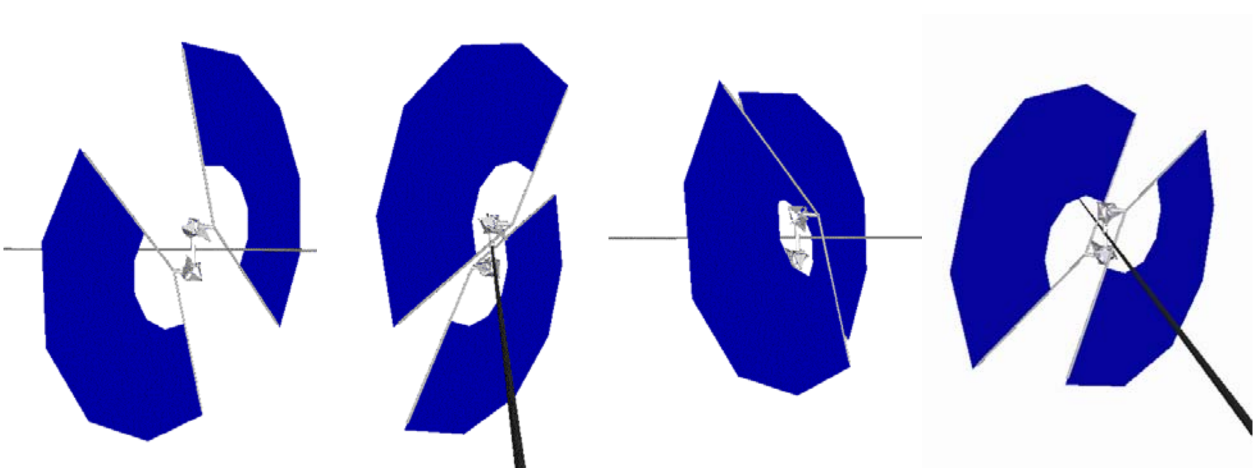


Figure 13: A unique arrangement of solar arrays mounted on Canfield joints enable constant solar tracking at any beta angle on a rotating facility with a minimum of bending loads across the array.

As shown in figures 10 and 11, liberal use of Canfield joints on the xGRF will enable solar tracking, high-data-rate communications, and even active propulsive stabilization to take place during the rotation of the facility. This, combined with the ease of spin rate change made possible through tether reeling, enables the deletion of the despun section and a tremendous overall facility simplification. These are the basic reasons to project a significant reduction in facility mass and cost.

Design Sequence

If the reader desires to design a variable-gravity facility in the manner described in the paper, the following design sequence is recommended, at least for a rough-order-of-magnitude, conceptual design:

1. Select the design artificial gravity level and the maximum angular rate of the facility in that condition. This information will determine the length of the moment arm from the center-of-mass of the facility to the habitat module in the retracted, spinning configuration. For instance, if it was desired to have a rotating facility with an angular rate of 4 rpm and an artificial gravity level of 1-g, the distance between the pressurized module and the center-of-mass would be:

$$r = \frac{a}{\omega_1^2} = \frac{9.81 m/s^2}{\left(4rpm(2\pi/60s)\right)^2} = 55.9m \tag{12}$$

2. Using the length of that moment arm and the countermass fraction, determine the total length of the tether in the retracted, spinning configuration. If the countermass represented 10% of the total facility mass and the pressurized module represented 90% of the total mass, then the total length of the tether in this configuration would be:

$$\ell_1 = \frac{r}{y} = \frac{55.9m}{0.1} = 559m \tag{13}$$

3. Using the initial angular rate of the facility (in its gravity-gradient configuration) and the maximum angular rate of the facility, calculate the retraction ratio. By assuming that the facility is initially in a gravity-gradient stabilized, radial configuration rotating the Earth at some orbit, the initial angular rate of the facility can be calculated. If we further make the assumption that the facility is in an essentially circular orbit, then the angular rate can be calculated from the orbital radius and the Earth's gravitational parameter:

$$\omega_0 = \sqrt{\frac{\mu}{(R_E + h)^3}} = \sqrt{\frac{398600 km^3/s^2}{(6378km + 500km)^3}} = 0.00111 rad/s \tag{14}$$

Based on these two angular rates, the retraction ratio can be calculated:

$$\frac{\ell_0}{\ell_1} = \sqrt{\frac{\omega_1}{\omega_0}} = \sqrt{\frac{4rpm(2\pi/60s)}{0.00111 rad/s}} = 19.45 \tag{15}$$

4. With the total retracted length and the retraction ratio, calculate the fully extended length—the length of the tether in the gravity-gradient configuration.

$$\ell_0 = \left(\frac{\ell_0}{\ell_1}\right)\ell_1 = 19.45(559m) = 10876m \tag{16}$$

These calculations can also be expressed in one equation:

$$\ell_0 = \frac{a}{y\sqrt{\omega_0 \omega_1^3}} = \frac{9.81 m/s^2}{0.1\sqrt{(0.00111 rad/s)(4rpm(2\pi/60s))^3}} = 10876m \tag{17}$$

In addition to these calculations, it is also important to keep track of the tip velocity of the tether at the facility end, insuring that should the tether be severed, that an immediate reentry of the facility does not result. For this case, the tip velocity at the facility end would be

$$v_1 = \frac{a_1}{\omega_1} = \frac{9.81 m/s^2}{4rpm(2\pi/60s)} = 23.4 m/s \tag{18}$$

From a 500-km circular orbit, a 23.4 m/s ΔV applied at the worst possible time in the tether's rotation would result in a drop to a 500 x 416 km orbit, insufficient to cause a reentry. Nevertheless, as a design proceeds, the designer should be careful to note the worst-case, severed-tether orbital condition and insure it is sufficient to preclude immediate reentry; angular rates greater than 2 RPM generally prevent this constraint from being violated.

Fractional Retraction Calculations

Often it will be desirable to design the facility to one condition, yet operate it in another. For instance, it is likely that the facility will be designed to a maximum acceleration of 1 g at some maximum angular rate, yet will operate most of the time at a fractional level of that acceleration, such as 1/6 g for investigations into lunar gravity effects.

Given some initial condition (fully-extended, gravity-gradient orientation) and some design condition (1-g operation at a maximum angular rate), what is the amount of retraction and the angular rate of an intermediate condition? Let us extend the use of the subscripts used previously—0 is the fully-extended, gravity-gradient configuration, 1 is the retracted, design configuration, and 2 is the intermediate, partially-retracted state.

To begin, let us assume that the intermediate level of acceleration (a_2) is based on some intermediate level of angular rate (ω_2) as well as on an intermediate length moment arm (r_2). The total length of the tether (ℓ_2) needed to support this acceleration level is based on the moment arm and on the countermass fraction.

$$a_2 = \omega_2^2 r_2 = \omega_2^2 \ell_2 y \tag{19}$$

The problem now is that we do not know either the angular rate or the total length necessary to generate these conditions. Fortunately, we can use the assumption that the facility has a constant angular momentum to relate angular rate and total length, as we did previously.

$$\frac{\omega_1}{\omega_2} = \left(\frac{\ell_2}{\ell_1}\right)^2 \tag{20}$$

$$\omega_2 = \frac{\omega_1 \ell_1^2}{\ell_2^2} \tag{21}$$

We then substitute for the result for intermediate angular rate and solve for the intermediate tether length:

$$a_2 = \frac{\omega_1^2 \ell_1^4 \ell_2 y}{\ell_2^4} = \frac{\omega_1^2 \ell_1^4 y}{\ell_2^3} \tag{22}$$

$$\ell_2 = \left(\frac{\omega_1^2 \ell_1^4 y}{a_2}\right)^{\frac{1}{3}} \tag{23}$$

This relationship is a bit cumbersome, and can be simplified by recognizing that the design acceleration (a_1) is calculated from the design angular rate, the design tether length and countermass fraction.

$$a_1 = \omega_1^2 \ell_1 y \tag{24}$$

We then substitute that definition into the intermediate length equation (Eq. 23):

$$\ell_2 = \ell_1 \left(\frac{a_1}{a_2}\right)^{\frac{1}{3}} \tag{25}$$

Let us redefine the intermediate acceleration (a_2) as a fraction (x) of the design acceleration (a_1).

$$\ell_2 = \ell_1 \left(\frac{a_1}{x a_1}\right)^{\frac{1}{3}} = \ell_1 x^{-\frac{1}{3}} \tag{26}$$

This simple relationship can be used to calculate the intermediate length given only the design length and the fraction of the design acceleration. This is particularly useful when the design acceleration is 1 g, since the fraction will then correspond to the fractional acceleration (i.e. 3/8 for Mars gravity, 1/6 for the Moon). Furthermore, we can also solve for the intermediate angular rate (ω_2) using the intermediate length (ℓ_2):

$$\omega_2 = \frac{\omega_1 \ell_1^2}{\ell_2^2} = \frac{\omega_1 \ell_1^2}{x^{-\frac{2}{3}} \ell_1^2} = \omega_1 x^{\frac{2}{3}} \tag{27}$$

Now both of these relationships can relate the intermediate angular rate and length to the design angular rate and length, based only on the fraction of acceleration between the design and intermediate conditions.

This relationship can also be used to show how quickly acceleration at the facility will be reduced as the tether is extended.

$$x = \left(\frac{\ell_1}{\ell_2}\right)^3 \tag{28}$$

The fractional acceleration will fall off as the cube of the ratio of design length to intermediate length. Hence, if a facility is operating in a 1-g condition and for some reason, a rapid spin-down is required, then the tether can be extended (releasing rotational energy, as was noted earlier) and the acceleration will plummet. When the tether is twice the design length, the acceleration level will be 1/8 of the design level. At 4 times the length, the acceleration is 1/64, and so forth. This capability for very rapid spin-down could be very important in accident or escape scenarios, as it is likely that spin-down through tether extension will be far more effective at reducing spin rate rapidly than a propulsive spin-down. The reason for this is that the propulsion system would linearly spin-down the facility as a function of the length of its burn (1/8 of the spin would require 7/8 of the burn duration) whereas the tether extension technique spins-down as a function of the cube of the extension ratio.

As was mentioned previously, it is important to consider the effect of tip velocity on the design in case of an accidental break of the tether. Does tip velocity increase in these fractional retraction conditions? Let us examine the intermediate value of tip velocity (v_2).

$$v_2 = \frac{a_2}{\omega_2} = \frac{x a_1}{x^{\frac{2}{3}} \omega_1} = v_1 x^{\frac{1}{3}} \quad (29)$$

We see from this expression that so long as the fractional acceleration desired is less than the design acceleration, the fractional tip velocity will be less than the design tip velocity. If the design tip velocity is within reentry constraints, then the fractional tip velocity will not be a problem.

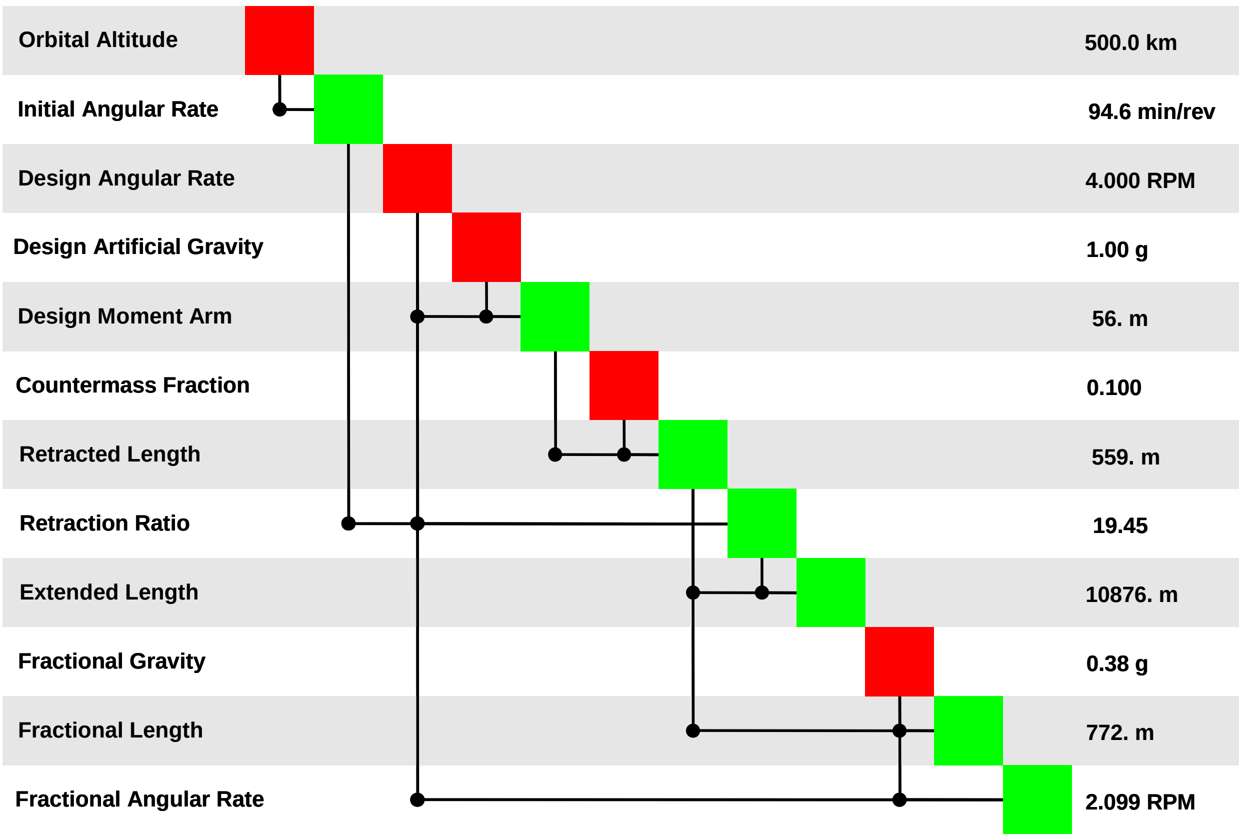


Figure 14: A design structure matrix showing the interactions between the different design variables and subsequent design calculations. The specific values displayed correspond to the design inputs given.

To summarize, the equations that enable one to start with some initial conditions and constraints, and construct a design for a variable-gravity facility based on these constraints, have been given herein. In an attempt to graphically depict the interactions and interplay between the design constraints and the results of the calculations, a visualization method called a "design structure matrix" is given. In the DSM, each input or calculation result is represented as a single box. The procedure begins with the box in the uppermost corner of the diagonal, and then proceeds step-by-step down the diagonal. Red boxes indicate primary inputs, and green boxes indicate calculations (dependent on previous calculations and primary inputs). For each calculation, the previous calculations or primary inputs upon which it depends are indicated by lines along the lower diagonal of the matrix. In this manner, the relationships between each calculation can be rather easily visualized.

Figure 15: Graphical depictions of the variable-gravity research facility concept, showing the inflatable pressurized module and the capsule hard-docked to it. The tracking solar arrays and tether deployer/reel, connected to the spent upper stage of the launch vehicle at the far end (serving as a countermass) are also shown.

Conclusions

The need for a variable-gravity research facility is clear—we must know how humans and equipment respond to the partial-gravity environment of the Moon and Mars. Previous designs for such facilities have been large and complicated because they have pursued fixed-moment-of-inertia concepts and propulsive spin rate change. These decisions have been made, in large part, on the complications to the facility from crew exchange, resupply, and pointing of power and communications equipment. Recent breakthroughs in pointing mechanisms allow the consideration of facilities that have no "de-spun" sections, which further allows the consideration of spin control based on a variable moment-of-inertia (tether reeling). The design equations have been described and show that the facility may be made small enough to launch on a single Delta 4 Heavy. These calculations also show that a large reduction in mass (and hopefully cost) over previous designs is feasible with these new technologies and techniques.

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