

**THE ARCHITECTURE OF ARTIFICIAL GRAVITY:  
ARCHETYPES AND TRANSFORMATIONS  
OF TERRESTRIAL DESIGN**

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**THE ARCHITECTURE OF ARTIFICIAL GRAVITY:  
ARCHETYPES AND TRANSFORMATIONS OF TERRESTRIAL DESIGN**

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**Abstract**

In artificial gravity, conformance to the hypothetical comfort zone does not guarantee an earth-normal environment, nor does it sanction design based on terrestrial norms. This paper begins by examining the range of gravity environments encompassed by the comfort zone. It compares Coriolis slope distortions with typical slopes in terrestrial architecture. It then offers a detailed analysis of the abnormalities inherent in relative motion through artificial gravity, using stair-climbing and material-handling as prototypical activities. The effect of Coriolis acceleration is such that it is impossible to design a stair for artificial gravity that meets the terrestrial design requirement of constant apparent slope at constant velocity in both the ascending and descending directions. Coriolis forces may also significantly reduce a person's effective lifting and carrying strength, even under partial gravity conditions. The only way to simulate a normal gravitational environment, with minimal Coriolis acceleration and rotational cross-coupling, is with a maximal radius of rotation. Where a large radius is not possible, habitat module orientation becomes important. The most comfortable orientation places the module axis parallel to the rotational axis. A deliberate, proactive approach to design may make artificial gravity more feasible by specifically planning for abnormal gravitational effects at small radii.

**Nomenclature**

Boldface indicates vector quantities; italics indicate scalar quantities; dots above indicate derivatives with respect to time:

- $X, Y, Z$  Inertial coordinates.
- $x, y, z$  Rotating coordinates.
- $x', y'$  Coordinates relative to observer.
- $\mathbf{i}, \mathbf{j}, \mathbf{k}$  Basis vectors in  $x, y, z$ .
- $\Omega$  Angular velocity of  $x, y, z$  relative to  $X, Y, Z$ .
- $\mathbf{R}, \dot{\mathbf{R}}, \ddot{\mathbf{R}}$  Position, velocity, acceleration relative to  $X, Y, Z$ .
- $\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}$  Position, velocity, acceleration relative to  $x, y, z$ .
- $R, V, A$  Magnitudes of  $\mathbf{R}, \dot{\mathbf{R}}, \ddot{\mathbf{R}}$ .
- $r, v, a$  Magnitudes of  $\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}$ .
- $t$  Elapsed time.
- $e$  Natural base = 2.71828...

- $\theta$  Position angle in  $x, y, z$ . ( $r, \theta$  are polar coordinates.)
- $\phi$  Velocity angle in  $x, y, z$ .
- $\alpha$  Velocity slope angle relative to  $x'$  axis.
- $\beta$  Rotation of  $x', y'$  axes to correct for Coriolis slope distortion.
- $\sigma$  Coriolis slope distortion when  $\beta = 0$ .

**Introduction**

Artificial gravity is often presented as a panacea for all of the ills associated with prolonged weightlessness. While extensive study has been devoted to the design of the artifact (structure, stability, propulsion, and so on), relatively little has been written about the design of the environment, from the point of view of an inhabitant living and moving within it. It has often been implied, and sometimes stated outright, that artificial gravity should permit the adoption of essentially terrestrial designs; the *artificiality* of the gravity has been downplayed. But saccharin is not sucrose, and centripetal acceleration is not gravity as we know it.

Human tolerance and adaptation to artificial gravity have been studied in centrifuges and slow rotation rooms. Boundary values for radius, angular velocity, and acceleration have been presented in various hard-edged comfort charts that characterize a set of values as being either in or out of a hypothetical "comfort zone". The tendency has been to reject any design that falls outside the zone, but to accept as "essentially terrestrial" any design within. Yet there are significant discrepancies between the comfort boundaries proposed by various authors, and all of them include conditions that hardly qualify as "earth normal". This suggests that the comfort boundaries are fuzzier than the individual charts imply, and that comfort may be influenced by task requirements and environmental design considerations beyond the basic rotational parameters.

Many proposals for artificial-gravity spacecraft have been developed over the past century, and it is difficult to imagine an unprecedented overall configuration. There is now a sufficient corpus of concepts to identify archetypes and enumerate certain critical aspects of configuration – for example: rings versus nodes; rigid spokes versus tethers; module axis longitudinal versus tangential versus radial; rotational axis parallel versus perpendicular to orbital axis; rotational axis inertial versus sun-tracking. Aside from the structural and dynamic considerations that may lead a designer to choose one configuration over another, each of these choices has consequences in habitat layout, relative motion of inhabitants, task design, comfort and efficiency.

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## Background

The apparent “up” vector for an observer in a rotating space station is defined by the vector sum of three accelerations:\*

$$\ddot{\mathbf{R}} = -\Omega^2 \mathbf{r} + 2\Omega \times \dot{\mathbf{r}} + \ddot{\mathbf{r}}$$

The first term  $[-\Omega^2 \mathbf{r}]$  is the centripetal acceleration: it represents the design gravity, and is the only term that is independent of the observer’s motion within the station. The second term  $[2\Omega \times \dot{\mathbf{r}}]$  is the Coriolis acceleration: it represents a distortion of apparent gravity, and is non-zero for any relative velocity with a non-zero projection in the plane of rotation. The third term  $[\ddot{\mathbf{r}}]$  is the observer’s relative acceleration: it is also a “distortion” of gravity, but its effect here is the same as for acceleration on earth, and should be somewhat familiar to the observer.

Another type of gravitational distortion arises from “cross-coupled” rotations. Rotating an object relative to the station, about an axis that is not aligned with the station’s axis, requires the application of a moment about a mutually perpendicular axis. Turning one’s head about a non-aligned axis causes vestibular disturbances and illusions of rotation that are roughly proportional to the vector cross-product of the angular velocities of the station and the head.<sup>1–2</sup>

## Comfort

Based on experiments in centrifuges and slow rotation rooms, researchers have developed various comfort charts for artificial gravity.<sup>3–8</sup> These charts specify boundary values for rotational parameters in an attempt to limit the adverse effects of Coriolis accelerations and cross-coupled rotations. They are succinct summaries of abstract mathematical relationships, but they do nothing to convey the look and feel of artificial-gravity. Consequently, there has been a tendency in many design concepts to treat any point within the comfort zone as “essentially terrestrial”, although that has not been the criterion for defining the zone. The defining criterion has been “mitigation of symptoms”, and authors differ as to the boundary values that satisfy it.

As iron filings reveal a magnetic field, so free-falling objects reveal a gravitational field. Perhaps a more intuitive way to compare artificial-gravity environments with each other as well as with earth is to observe the behavior of an object when dropped from a certain height or launched from the floor at a certain velocity. Such a comparison is suggested in Figures 1 and 2. Figure 1 shows for earth-normal gravity the effect of hopping vertically off the floor with an initial velocity of 2 meters per second and of dropping a ball from an initial height of 2 meters. The “hop” and the “drop” each trace vertical trajectories; the “hop” reaches a maximum height of 0.204 meters (8.03 inches), indicated by a short horizontal line; the “drop” is marked by dots at 0.1-second intervals. Figure 2 shows a typical comfort chart for artificial gravity, after that of Hill and Schnitzer<sup>3</sup>, surrounded by five similar “hop and drop” dia-

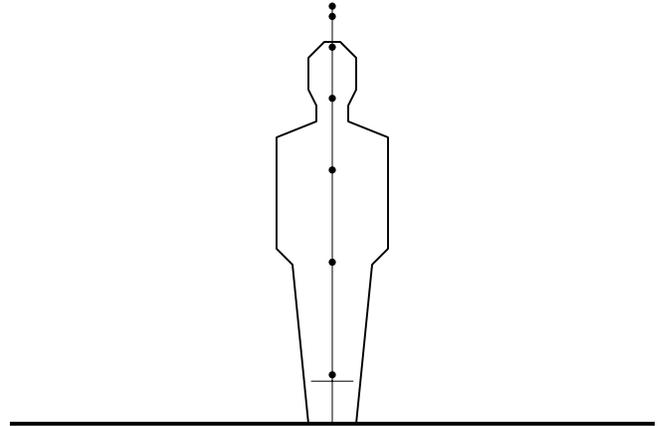


Figure 1: Earth-Normal Gravity

grams – one for each boundary point of the comfort zone. These diagrams reveal certain features of the comfort boundaries:\*\*

- Large radius – points 5 and 1: Artificial gravity becomes increasingly “normal” as the radius of rotation approaches infinity.<sup>9</sup> The trajectory of a dropped object depends only on the radius of rotation and the initial height of the object. Thus, the drops at points 5 and 1 follow congruent paths, although the drop at 5 is much slower due to the low gravity. (The dots are spaced at 0.1-second intervals.) The trajectory of a thrown object is influenced by the ratio of its initial relative velocity to the tangential velocity (rim speed) of the station. Thus the hop at point 5, besides being much higher (due to the low gravity), is also more distorted than at point 1 due to the lower tangential velocity of the station. Point 1 is the most “earth-normal” point on the chart; point 5 approaches “normal” for a planetesimal or asteroid.
- Earth gravity – points 1 and 2: Earth-force does not imply earth-normal. Although both points represent 1-g environments, both the hop and the drop are more distorted at point 2, due to the smaller radius and lower tangential velocity.
- High angular velocity – points 2 and 3: The upper limit of angular velocity is determined by the onset of motion sickness due to cross-coupled rotations. At this boundary, reducing the radius reduces the centripetal acceleration and tangential velocity as well. As judged by the “twisting” of the apparent gravitational field, point 3 is the least normal point in the comfort zone.
- Low tangential velocity – points 3 and 4: For a given relative motion, the ratio of Coriolis to centripetal acceleration increases as tangential velocity decreases. Between points 3 and 4 it is constant, and the hops at these points have similar shapes, though the hop at point 4 is larger due to the lower acceleration. The drop at point 4 is straighter due to the larger radius.
- Low gravity – points 4 and 5: Although the centripetal acceleration at these points is equal, the gravitational environ-

\* This assumes that the station is unpropelled, that angular velocity is constant, and that gravity gradients are negligible.

\*\* The diagrams were plotted with an artificial-gravity simulation program developed by me on Apollo computers.

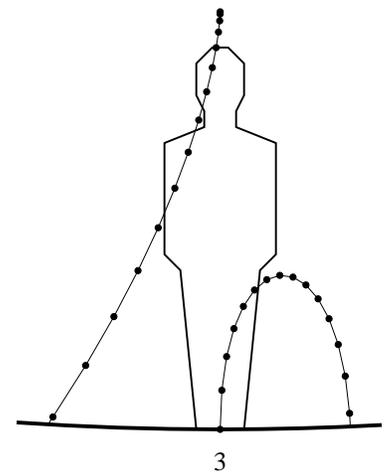
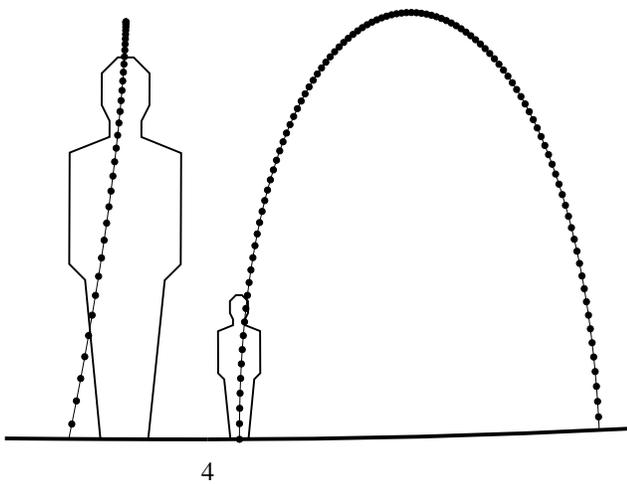
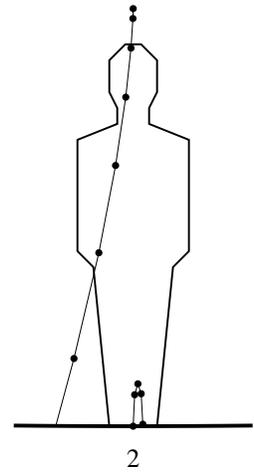
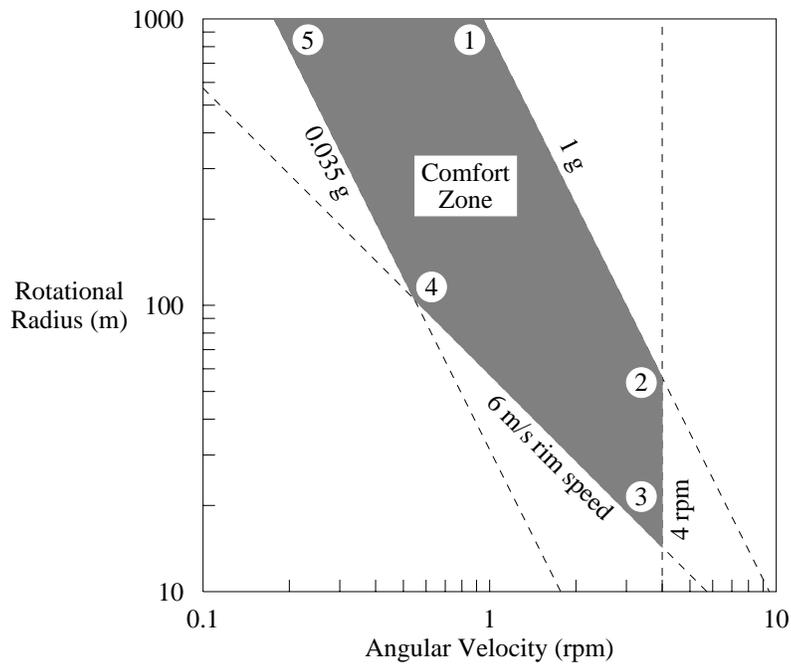
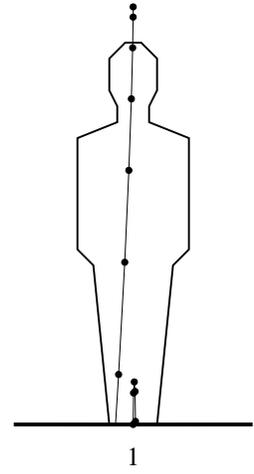
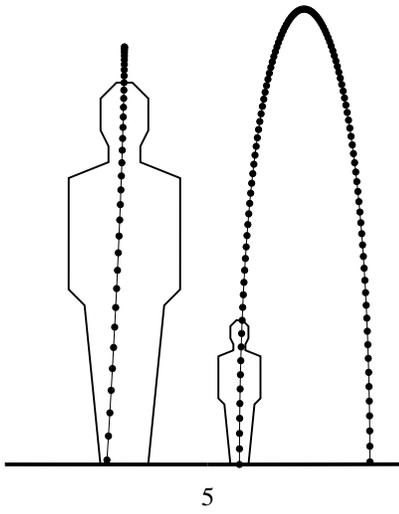


Figure 2: Artificial Gravity and the Comfort Zone

ment is less distorted at point 5 due to its larger radius and higher tangential velocity.

Evidently, the comfort zone encompasses a wide range of environments, many of them substantially non-terrestrial. Conformance to the comfort zone does not guarantee an earth-normal gravity environment, nor does it sanction “essentially terrestrial” design.

### Apparent Slope

The Coriolis acceleration represents a gravity component that is neither intended nor expected in a normal gravitational environment. For circumferential relative motion, the Coriolis and centripetal accelerations are aligned, and the net effect is a change in apparent weight but not in apparent slope. However, if there is a radial component of relative velocity, then the Coriolis acceleration produces a misalignment of the centripetal and total accelerations. The angle between these vectors constitutes an apparent slope that depends on the observer’s speed and direction of motion.

Figure 3 is a vector diagram showing the relationship between relative velocity, centripetal acceleration, Coriolis acceleration, and the total of these accelerations. For velocity  $v$  at angle  $\alpha$  measured from the  $x'$  axis, the slope distortion due to Coriolis acceleration is:

$$\sigma = \arctan\left(\frac{2\Omega v \sin(\alpha)}{2\Omega v \cos(\alpha) + \Omega^2 r}\right) \quad (1)$$

If  $\Omega$  and  $v$  are both non-zero, this can be written as:

$$\sigma = \arctan\left(\frac{\sin(\alpha)}{\cos(\alpha) + \frac{\Omega r}{2v}}\right) \quad (2)$$

In assessing the significance of Coriolis acceleration and apparent slope, it is helpful to consider a few guidelines from earthly design. Table 1 has been culled from the BOCA National Building Code<sup>10–11</sup>, Templer<sup>12</sup>, derivation, and personal observation.

People are generally poor judges of slope, and tend to overestimate the steepness of hills. An incline of  $4^\circ$  feels much steeper to a pedestrian than it looks on paper, and it is common practice in site design to exaggerate the vertical scale – to facilitate drafting of drainage details as well as to convey the “feel” of the terrain. After estimating the Coriolis distortion for an artificial-gravity environment, comparison with slope values such as those in Table 1 may indicate whether the architectural paradigm should be terrestrial, or naval, or something else entirely.

Many artificial-gravity design proposals call for accelerations of less than  $1g$  – usually because of restrictions on radius and angular velocity imposed by economics and the comfort zone. While reducing the angular velocity reduces both the Coriolis and centripetal accelerations, it *increases* the ratio of Coriolis to centripetal:

$$\frac{2\Omega v}{\Omega^2 r} = \frac{2v}{\Omega r} \quad (3)$$

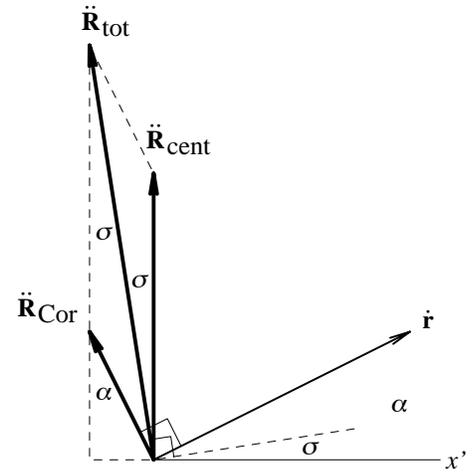


Figure 3: Coriolis Slope Distortion

Typical Slopes in Terrestrial Architecture	
Condition	Slope
Maximum slope for residential stairs (8.25" riser, 9" tread)	42.5°
Maximum slope for public stairs (7" riser, 11" tread)	32.5°
Maximum slope for means-of-egress ramps for healthy persons (1:8)	7.1°
Maximum slope for means-of-egress ramps for hand-capped persons (1:12)	4.8°
Slope at which warning signs are posted on some highways (7% grade)	4.0°
Maximum wash of stair tread (1:60)	1.0°
End point rotation of simply-supported, uniformly-loaded floor beam under maximum deflection ( $deflection_{max} = span/360$ )	0.5°
Minimum slope for 2.5" sewage drain (1/4 inch per foot)	1.2°
Minimum slope for 8" sewage drain (1/16 inch per foot)	0.3°

Table 1: Typical Slopes in Terrestrial Architecture

The larger this ratio, the larger the slope distortion in equation (2). Thus for any given radius, while reducing  $\Omega$  ameliorates problems associated with rotational cross-coupling (such as dizziness, ataxia, and nausea), it exacerbates gravitational distortion and velocity-dependent apparent slopes.

## Stair and Ladder Design

Stairs, ladders, and elevators appear in virtually all multi-level artificial-gravity designs. They embody the designer's concept of the gravity environment. As stair-climbing is a familiar daily activity for most people, it is a good subject for testing that concept. Stair-climbing is emblematic of motion through gravity.

Templer cites the following facts with regard to stair hazards: <sup>12</sup>

- In the United States, falls are the second largest cause of accidental death, outranked only by automobile accidents. Falls cause more than twice as many deaths as drowning or fires and burns.
- Steps and stairs are the most dangerous element in the home in the United States, the United Kingdom, and the Netherlands.
- In Japan, accidental falls outrank traffic accidents as a cause for hospital treatment.

“These figures indicate that stairs as designed and built (but not necessarily as they might be designed and built) are some of the most dangerous artifacts in our environment, and they also suggest that as much research attention should be paid to stairs as to fires.” In traversing a stair, any violation of expectations may be hazardous. Nelson showed that, in normal ascent, the clearance between the stair and the shoe may be as small as 3/8 inch; a misreading of the stair to this extent is enough to cause an accident. <sup>13</sup>

In 1976, O'Neill and Driggers used a cursory examination of stair-climbing to comment on the observable effects of rotating environments. <sup>14</sup> For a “brisk rate of climb of about 1 foot per second” in 1-g environments rotating between 1 rpm and 3 rpm, they calculated ratios of Coriolis force to centripetal weight ranging from 0.01 to 0.03, and corresponding “angles of lean” ranging from 0.6° to 1.8°. They maintained that “such small angles generally can be considered to be negligible,” but the validity of that assertion is questionable. In particular, a tread slope (wash) of 1.8° considerably exceeds the 1° maximum cited by Templer\*. Furthermore, an ascent rate of 1 foot per second is only average\*\*; “brisk” ascent (or descent) would result in larger “angles of lean”.

In an artificial-gravity environment, “ascent” is defined as motion from a greater to a lesser radius. If the rate of ascent (relative velocity) is constant, then this motion results in an

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\* Actually, O'Neill and Driggers seem to have rounded-up significantly in calculating the “angle of lean”. Including only the tangential component of Coriolis acceleration associated with radial motion, the formula is:

$$\text{lean} = \arctan\left(2\Omega v_r / \Omega^2 r\right)$$

If  $\Omega^2 r$  is 32.2 ft/s<sup>2</sup> (1 g),  $v_r$  is 1 ft/s, and  $\Omega$  ranges from 0.105 to 0.314 rad/s (1 to 3 rpm), then the lean angle ranges from 0.4° to 1.1°. Though significantly less than 1.8°, it still exceeds the 1° maximum considered acceptable in terrestrial design.

increasing Coriolis / centripetal ratio and distortion of gravity. According to equation (3), the only way to avoid this increasing distortion is to decrease relative velocity in proportion to radius – an unreasonable expectation.

There have been several proposals for radially-oriented “high-rise” rotating space stations fashioned from space station habitability modules, shuttle external tanks, or similar structures. <sup>15–19</sup> The crew members' daily routine would involve vertical circulation between decks operating at a wide range of gravity levels. A high deck could operate at lunar gravity (0.17 g) while a lower deck operates at Mars gravity (0.38 g), allowing variable-gravity research to be conducted at a range of gravity levels simultaneously. Snead <sup>18</sup> proposes that regular stair-climbing between decks would provide exercise to maintain the health of the crew members.

Figure 4 illustrates one “spoke” of such a station with a rotation rate of 2 rpm. In the center is a schematic section of the structure. At the left are diagrams for lunar-normal and Mars-normal gravity. At the right are diagrams for lunar-intensity and Mars-intensity artificial gravity at 2 rpm. When these are compared to the earth-normal gravity shown in Figure 1, there is cause to wonder whether typical terrestrial stairs are the best choice for climbing between decks in this station – especially in the low-gravity region near the top. Certainly stairs could be built, but it seems doubtful that they would function as intended. Given the low gravity and the large distortion, a mode of transport that readily accommodates the hands as well as the feet – such as a ladder or “fireman's pole” – may be a better choice.

On earth, stairs are generally designed to maintain a constant slope (rise over run) with respect to a level surface. In fact, this is a requirement for safe stair design, and the building code sets strict limits on the dimensional variation allowed within a flight of stairs. Furthermore, this slope is independent of the rate at which the stairs are traversed. It is reasonable to expect a constant velocity, but this has no bearing on the design.

Noordung's *Wohnrad* concept included stairs spiraling from the rim of the station toward the central hub. <sup>20</sup> If such stairs are designed to maintain a constant slope relative to centripetal acceleration, and Coriolis acceleration is ignored, the curve is described by the formula:

$$r(\theta) = \frac{r_{max}}{e^{\left(\frac{\text{rise}}{\text{run}} \theta\right)}} \quad (4)$$

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\*\* According to Templer, the prediction equation for rate of ascent that best fits the available data is:

$$v_r = 76.98 + 2.106 \text{ riser} - 2.543 \text{ tread}$$

where *riser* and *tread* are measured in inches and  $v_r$  is the rate of vertical ascent in feet per minute. For a 6” riser and 12” tread, this predicts an average rate of ascent of 59.1 feet per minute, or 0.985 feet per second.

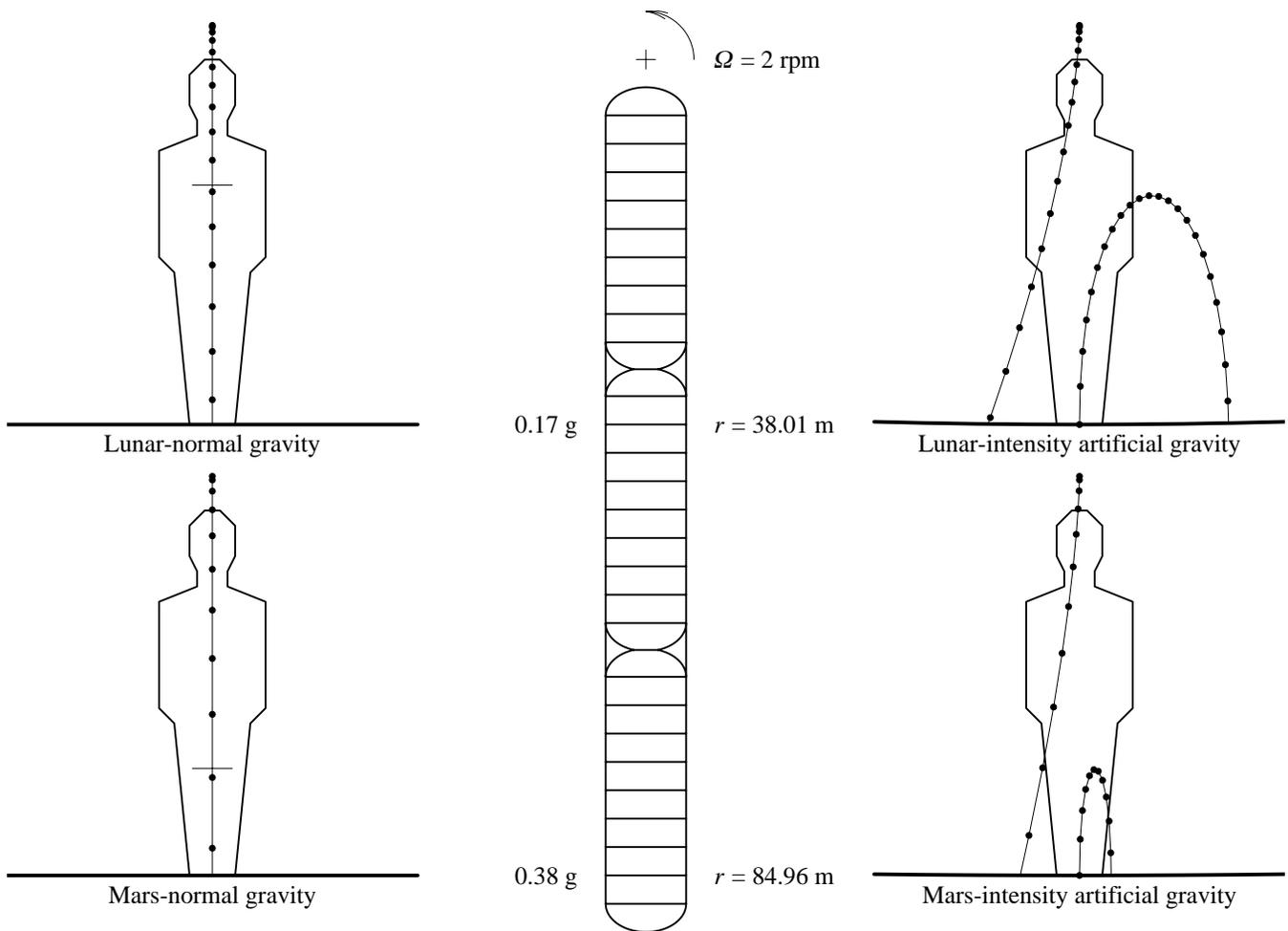


Figure 4: Gravity Levels in a Radially-Oriented Station

In parametric form, assuming  $v$  is constant speed of motion along the curve, and  $r$  and  $\theta$  are functions of time  $t$ :

$$\alpha = \arctan\left(\frac{\text{rise}}{\text{run}}\right) \quad (5)$$

$$r = r_{\max} - vt \sin(\alpha) \quad (6a)$$

$$\theta = \cot(\alpha) \ln\left(\frac{r_{\max}}{r_{\max} - vt \sin(\alpha)}\right) \quad (6b)$$

Figure 5 shows an example of such a curve. It may be an adequate form for a stair when the radius is large, but its failure to account for Coriolis acceleration may render it inadequate at smaller radii.

Let us examine the severity of the gravitational distortion and the possibility of adapting the curve to account for it. Figure 6 is a vector diagram showing the relationship between relative velocity, centripetal acceleration, Coriolis acceleration, and the total of these accelerations for ascending a stair to the east (prograde). Figure 7 shows the relationship for descending to the west (retrograde). (The “total” acceleration in these figures is not the grand total of all accelerations, since it does not

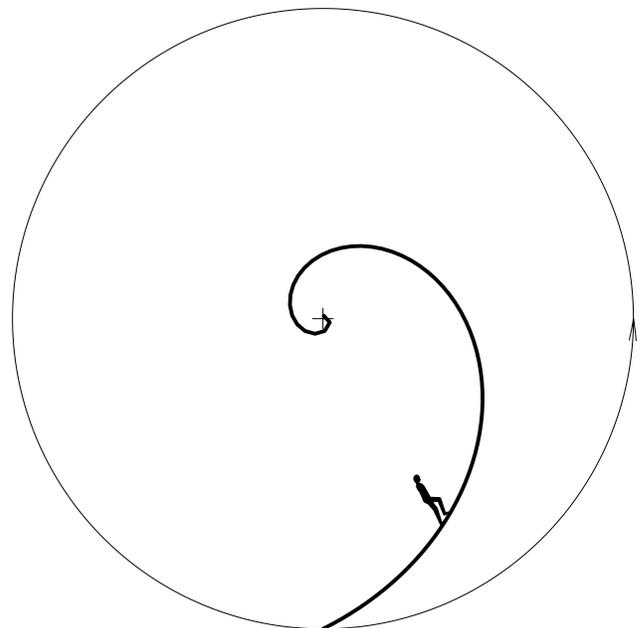


Figure 5: Spiral Stair with Constant Slope

include the relative acceleration  $[\ddot{\mathbf{r}}]$  that arises from the curvature of the path itself. That missing term will be discussed later.) The  $x'$  axis is now rotated to the perceived horizontal, perpendicular to the total acceleration. The angle  $\alpha$  is the intended slope of the relative velocity, in the range  $\pm\pi$  measured from the  $x'$  axis. The angle  $\beta$  is the rotation of the  $x', y'$  axes required to compensate for the Coriolis acceleration. From trigonometry and the law of sines:

$$\begin{aligned} \beta &= \arcsin\left(\frac{A_{Cor}}{A_{cent}} \sin(\alpha)\right) \\ &= \arcsin\left(\frac{2v}{\Omega r} \sin(\alpha)\right) \end{aligned} \tag{7}$$

The modified curve is then defined by a set of differential equations:

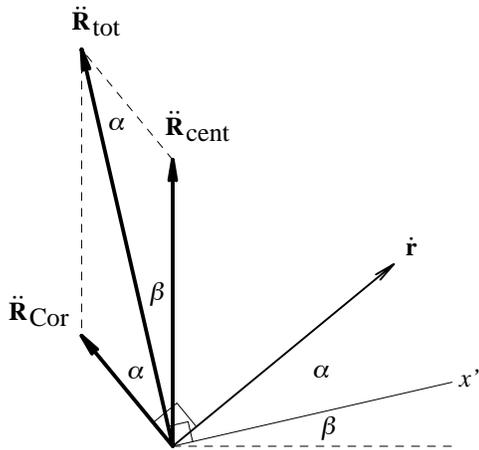


Figure 6: Slope Adjustment for Prograde Ascent

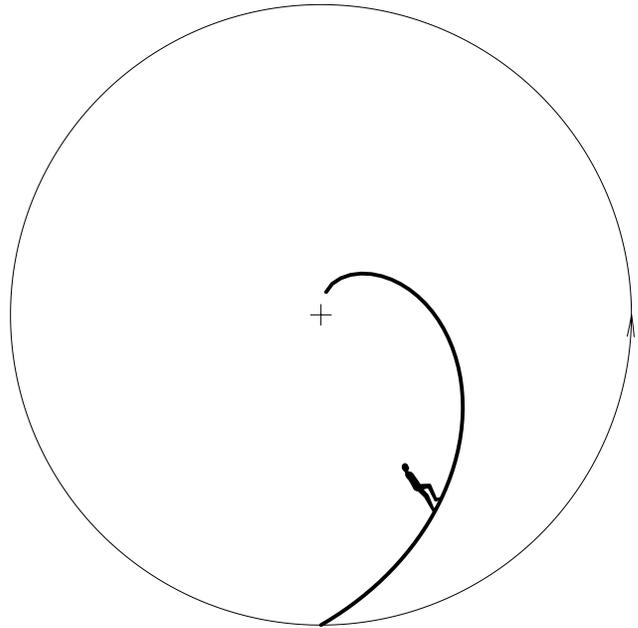


Figure 8: Modified Spiral for Prograde Ascent

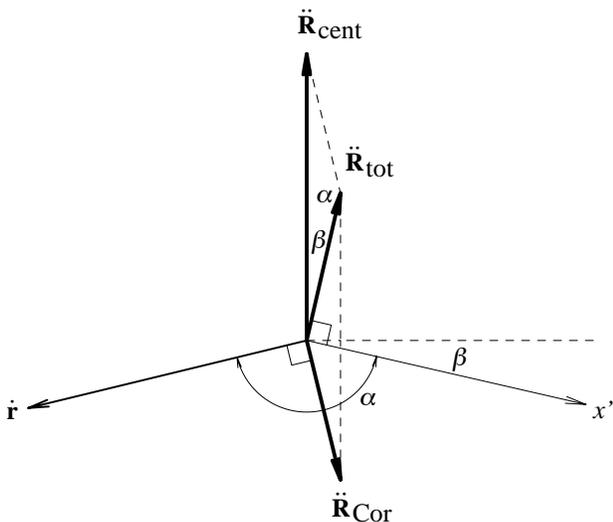


Figure 7: Slope Adjustment for Retrograde Descent

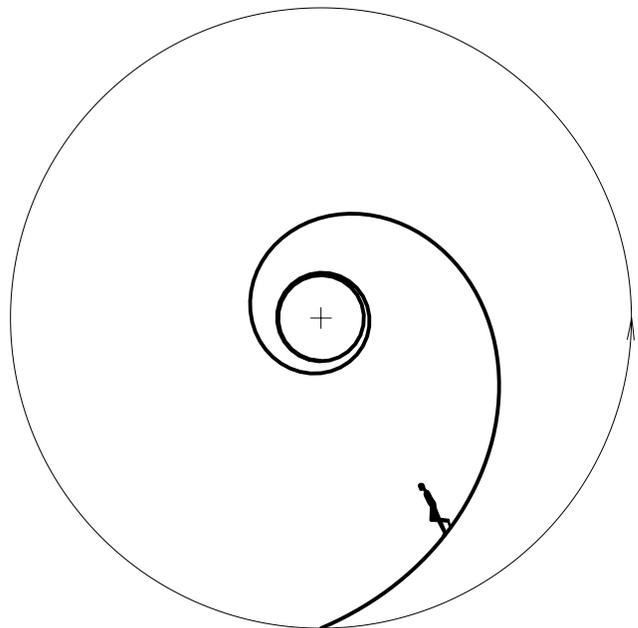


Figure 9: Modified Spiral for Retrograde Descent

$$\dot{r} = -v \sin\left(\alpha + \arcsin\left(\frac{2v}{\Omega r} \sin(\alpha)\right)\right) \quad (8a)$$

$$\dot{\theta} = \frac{v \cos\left(\alpha + \arcsin\left(\frac{2v}{\Omega r} \sin(\alpha)\right)\right)}{r} \quad (8b)$$

$$r(0) = r_{max} \quad (8c)$$

$$\theta(0) = 0 \quad (8d)$$

Figures 8 and 9 show the modified curves for ascending and descending.\* Several features are worth noting:

- The curves for ascending and descending are different. The differences are small when the radius is large, but become significant as the radius decreases. This means that, at small radii, it is impossible to construct a single stair for both ascending and descending that satisfies the “earth-normal” requirement for constant apparent slope at constant velocity.

- For prograde motion, the following conditions hold:

$$|\alpha| < \frac{\pi}{2}$$

$$r \geq \frac{2v}{\Omega} |\sin(\alpha)|$$

At smaller radii, the argument to the arcsin function is outside the range  $\pm 1$ , and no solution exists. The curve simply disappears before reaching the center.

- For retrograde motion, these conditions hold:

$$|\alpha| > \frac{\pi}{2}$$

$$\arcsin(\sin(\alpha)) = \pi - \alpha \quad \text{if } \alpha > 0$$

$$\arcsin(\sin(\alpha)) = -\pi - \alpha \quad \text{if } \alpha < 0$$

$$r \geq \frac{2v}{\Omega}$$

As the curve approaches the center,  $\dot{r}$  approaches zero. The spiral approaches a circle at a finite distance from the center, and never reaches the center. An observer descending from this radius would have the Escher-esque experience of going down stairs without getting any lower. The key to this apparent paradox is that retrograde motion at small radii essentially cancels the rotation of the environment. The observer is weightless, and there is no “up” or “down”.

For both ascending and descending, the minimum radius of the spiral increases with the square root of the design radius. (The minimum radius is inversely proportional to  $\Omega$ ; the design radius is inversely proportional to  $\Omega^2$ .) Furthermore, the modi-

fied curves become substantially different well before the mathematically minimum radius is reached.

This analysis is incomplete, since it does not consider the contribution of the relative acceleration [ $\ddot{\mathbf{r}}$ ] associated with the curvature of the path. At large radii, the curvature is slight, and this acceleration is small even compared to the Coriolis. But it increases as the radius decreases, and becomes significant at small radii. (For circumferential motion, the magnitude of this acceleration is  $v^2/r$ . For spiral motion, it is harder to determine.)

If the speed  $v$  is constant, then the acceleration vector  $\ddot{\mathbf{r}}$  must be perpendicular the velocity vector  $\dot{\mathbf{r}}$ . Thus it is aligned with the Coriolis vector, and either adds to or subtracts from it depending on whether the relative motion is prograde or retrograde. (The relative acceleration is always directed toward the interior of the spiral, whereas the Coriolis is directed in or out depending on the direction of motion.) At first, it might appear possible to account for this additional acceleration by modifying the system of differential equations. “Abandon all hope, ye that enter here:”

$$\phi = \theta + \frac{\pi}{2} + \beta + \alpha$$

$$\dot{\mathbf{r}} = v(\cos(\phi)\mathbf{i} + \sin(\phi)\mathbf{j})$$

$$\ddot{\mathbf{r}} = v\dot{\phi}(-\sin(\phi)\mathbf{i} + \cos(\phi)\mathbf{j})$$

$$a = |\ddot{\mathbf{r}}| = v\dot{\phi}$$

$$= v(\dot{\theta} + \dot{\beta})$$

$$\beta = \arcsin\left(\frac{A_{Cor} \pm a}{A_{cent}} \sin(\alpha)\right)$$

$$= \arcsin\left(\frac{2\Omega v \pm v(\dot{\theta} + \dot{\beta})}{\Omega^2 r} \sin(\alpha)\right) \quad (9a)$$

$$\dot{r} = \quad (9b)$$

$$-v \sin\left(\alpha + \arcsin\left(\frac{2\Omega v \pm v(\dot{\theta} + \dot{\beta})}{\Omega^2 r} \sin(\alpha)\right)\right)$$

$$\dot{\theta} = \quad (9c)$$

$$\frac{v \cos\left(\alpha + \arcsin\left(\frac{2\Omega v \pm v(\dot{\theta} + \dot{\beta})}{\Omega^2 r} \sin(\alpha)\right)\right)}{r}$$

$$r(0) = r_{max} \quad (9d)$$

$$\theta(0) = 0 \quad (9e)$$

\* Solutions to the differential equations were plotted with Mathematica® NeXT release 2.1, NeXT system release 3.0.

These modified equations defy solution (by me, anyway): adding the relative acceleration increases  $\beta$ , which increases the curvature of the spiral, which increases the acceleration, which increases  $\beta$ , and so on. The process does not seem to converge.

Having apparently hit a mathematical dead-end, we can happily abandon this analytical overkill with a clear conscience, knowing that it leads nowhere. For even if the equations could be solved, they would lead to a stair shape that was perfectly adapted for only one particular velocity and direction – clearly not an acceptable general solution.

The moral is that it is impossible to design away the gravitational distortions inherent in rotating environments. They can be kept arbitrarily small only by keeping the radius sufficiently large. Where radius is limited, the Coriolis accelerations and cross-coupled rotations may feel like the pitch, roll, and yaw of a ship at sea. Sailors have been adapting to these motions for centuries, aided by naval architects who have designed environments to accommodate them – for example, by providing narrow stairs with two easily-grasped handrails. Nevertheless, even naval architecture does not offer a perfect paradigm. In artificial gravity, the disturbances have a definite predictable relationship to the observer’s relative motion; this offers the designer an opportunity to influence the orientation of these disturbances relative to the observer – whether they occur in the coronal, sagittal, or transverse plane. And, the magnitude of artificial gravity may be substantially different than 1 g – a situation not faced by naval architects (to date).

If one can not avoid gravitational distortion, then one must design for it. Figure 10 shows the acceleration of a person ascending a straight ladder aligned on a radius in a rotating environment. Figure 11 shows the situation as that person perceives it. For any uniform linear motion in a rotating environment, the variations in both apparent slope and apparent weight are described by catenary curves.<sup>9</sup> (This apparent curvature applies to flat floors and straight stairs, as well as to ladders.) The ladder should be oriented so that the user is pressed into this curve from above, and not pulled away from below or sideways. Since Coriolis accelerations occur only in the plane of rotation, the plane of the ladder should be perpendicular to that plane. Furthermore, since the direction of the Coriolis acceleration reverses with the direction of motion, the ladder should be accessible from both sides. A user would find it most comfortable to ascend on the west side (as shown), and descend on the east side.

In a similar vein, an elevator car designed to move along a radial path should provide braces or restraints for the passengers to lean against, or should pivot to align itself with the total acceleration vector. If the passenger compartment is designed to pivot freely about an axis above its center of mass, then its alignment with the acceleration will be self-correcting.

**Material Handling**

Newton’s Laws do not distinguish between stair-climbing and other motion-related tasks. The gravitational effects encountered in climbing a set of stairs manifest themselves in virtually all activities. The differences are matters of quantity rather than quality.

Chaffin directed a study of human strength predictions for two-handed lifting, pushing, and pulling tasks under various conditions.<sup>21</sup> The study was based on computer simulation, using a biomechanical model and statistical data for body size

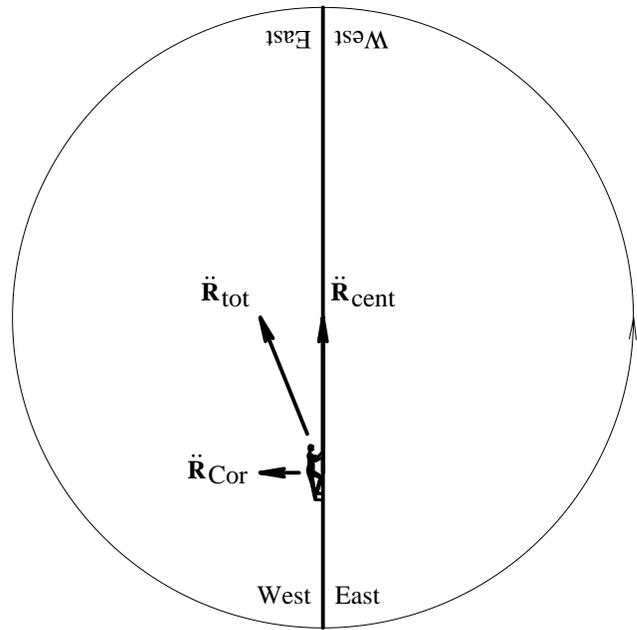


Figure 10: Climbing a Ladder in Artificial Gravity

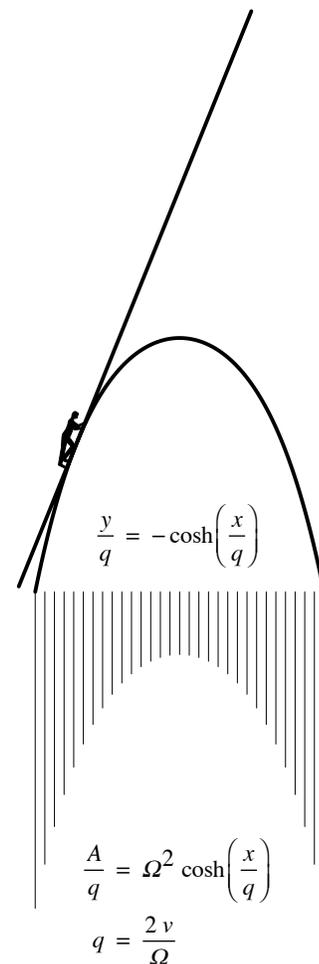


Figure 11: Apparent Slope and Strength of Gravity

and mass, muscle strength, and range of motion. In addition to raw muscular exertion, the strength predictions considered the effects of posture, balance, and stability: while low gravity reduces the weight being lifted, it also reduces the weight of the person doing the lifting. “It can therefore be shown that for some tasks performed under reduced gravity conditions, man’s strength is increased, but for others, it is decreased.” A person’s mass-lifting ability is *not* inversely proportional to the strength of gravity.

Gravity levels of 0.2 g and 1.0 g were simulated to provide a comparison of lunar and earth environments. A condition of 0.7 g was also simulated, with the thought that it might be applicable to a spinning space station. However, no radius or angular velocity were specified, so the simulation apparently did not consider the effects of Coriolis accelerations and cross-coupled rotations. Its predictions are valid only for spinning space stations of large radius and low angular velocity in which these effects are negligible.

In contrast, Stone proposes that acceptable Coriolis forces in material handling may be as high as 25% of the centripetal forces.<sup>6</sup> If the weight being handled is near the handler’s strength limit (based on experience in normal gravity), a Coriolis factor of 25% may be more than enough to exceed his grip, strain a muscle, or knock him off balance. While it may be possible for humans to adapt to work in such an environment, the cost of adaptation is often decreased performance. This must be weighed against the cost of increased radius and decreased angular velocity. Stone writes that “it is therefore desirable to determine the smallest radius and rate of rotation at which acceptable performance and habitability may be attained.” A carefully planned, deliberately non-terrestrial work environment may permit a smaller radius, by orienting relative motion so as to minimize gravitational distortion and provide workers with the best mechanical advantage in overcoming it.

### Choosing Among Archetypes

Many artificial-gravity spacecraft concepts have been published over the past century, beginning with Konstantin Tsiolkovsky’s early work in the 1890’s; a comprehensive bibliography would exceed the length allowed for this paper. Rather than discuss the strengths and weaknesses of myriad individual concepts, it is more useful to group them according to one or more archetypal characteristics, and discuss the relative merits of those characteristics. Schultz, Rupp, Hajos, and Butler,<sup>22</sup> and Capps, Fowler, and Appleby<sup>23</sup> adopted similar approaches in discussing the evolution and optimization of manned Mars vehicles. While those papers focused on inanimate engineering concerns (such as power, propulsion, mass, and stability), this paper is aimed at issues related to habitability (notwithstanding its foray into physics and differential equations).

Few people would deny that humans are substantially different than machines, having been endowed with consciousness, intelligence, culture, expectation, emotion, and irrationality. It should not be surprising to find that a spacecraft optimized for human habitation may be substantially different than one optimized for automation and remote sensing. If space is really to be humanity’s high frontier, then habitability must be weighed against other engineering concerns at the outset, and not left as an afterthought. Technology and economics may preclude luxury, but if basic creature comforts must be compromised, it should at least be an informed decision, cognizant of the consequences.

### Rotational Radius

This paper has focused on one aspect of habitability: the physical forces that impinge on people in motion through an artificial-gravity environment. In this domain, the single most important variable is radius:

- The larger the radius, the better. For any choice of centripetal acceleration (gravity level), increasing the radius increases the tangential velocity and decreases the angular velocity. Both of these effects lead to a more natural gravity environment.
- A very large radius is (probably) best accomplished with tethers rather than rigid (massive) spokes.
- A large radius precludes toroidal habitats, unless the required floor area and volume are also very large. A moderate volume stretched around a large circumference results in a cross section that is too skinny to be efficiently inhabited. (Too much of it is given over to circulation.)
- A non-toroidal rotating habitat requires a counter mass. Either the habitat must be partitioned in to two or more nodes, or other non-habitable masses must be identified and isolated.

### Module Axis Orientation

If a large radius is not possible, then the orientation of the habitat’s longitudinal axis relative to the spin axis becomes important. The following remarks assume a circular cylindrical module:

**Radial.** While there may be some advantages to a radial (vertical) orientation, it appears to be the least comfortable:

- A range of gravity levels are provided simultaneously, which may be useful in some types of research.
- Occupants must climb through a range of gravity levels on a regular basis. Coriolis accelerations are unavoidable.
- Equipment, tools, and miscellaneous supplies are difficult to carry on ladders, and even on stairs if the gravity is too distorted.
- Stairs consume a large portion of the plan.
- The possibility of orthostatic intolerance on descending to a greater gravity level exacerbates the danger of falling.
- Arrangement of workstations around the perimeter of a circular plan requires workers to swivel around an axis perpendicular to the habitat’s rotation, leading to cross-coupled rotations and consequent vestibular disturbances.
- A circular plan accommodates windows in any direction.
- The vertical internal arrangement is not compatible with current space station design.

**Tangential.** The tangential (or circumferential) orientation appears to offer intermediate comfort. It is mandated by toroidal designs:

- The range of gravity levels is limited.

- Vertical circulation is typically limited to three decks or less.
- The primary circulation is circumferential, leading to weight gains or losses depending on the direction of travel. Coriolis accelerations are unavoidable.
- Floors and work surfaces must be curved, or apparent slopes must be tolerated. Module joints must be angled to prevent the accumulation of large apparent slopes.
- Curved floors and work surfaces require wedge-shaped or trapezoidal equipment racks.
- Arrangement of workstations along the sides promotes cross-coupled head rotations – whether back-and-forth or up-and-down.
- Equipment racks slide axially, without Coriolis effects.
- Windows arranged along the sides view parallel to the rotation axis. The scene appears to rotate about the center of the window. Sunlight is continuous over the rotational period of the habitat (without alternation from light to shadow), although the beam “orbits” the room if the window is pointed obliquely toward the sun.

**Axial.** The axial orientation (module axis parallel to rotation axis) offers the most comfortable gravitational environment:

- The range of gravity levels is limited.
- Vertical circulation is typically limited to three decks or less.
- The primary circulation is parallel to the rotation axis, free from Coriolis accelerations.
- Floors and work surface are flat in the length dimension. Curvature in the width dimension may not be necessary if the module width is small relative to the rotational radius.
- Arrangement of workstations along the sides permits up-and-down head rotations without cross-coupling. Back-and-forth head rotations still cross-couple, however.
- Equipment racks slide tangentially, encountering some Coriolis force. However, these forces should be well within the load capacity of the racks.
- Windows arranged along the sides view tangential to the rotation. The scene appears to pan or rotate vertically. Sunlight is stroboscopic over the rotational period of the habitat unless the rotation axis is aligned with the sun.
- The horizontal internal arrangement is compatible with current space station design.

### **Conclusion**

Providing artificial gravity will undoubtedly increase the cost of spacecraft. If technological or economic constraints mandate an imperfect gravity environment, then it is all the more important to design that environment without naive assumptions of earth-normalcy, so as to minimize the costs of adaptation, retraining, and reduced productivity. Conversely, a

proactive approach to design may make artificial gravity more feasible by specifically planning for abnormal gravitational effects at small radii.

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